# Individual Choice under Uncertainty

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## Summary:

The first part of this paper gives an overview of the dominating approach within economic theory on individual decision making under uncertainty or risk — the expected utility (EU) theory. The theory has increasingly been criticised, and some of the empirical violations of the theory are reviewed. In spite of a number of paradoxes, and the descriptive and predictive difficulties of the EU theory, it remains the dominating approach within economic theory. This paper presents and discusses two alternative, non-expected utility approaches, which fit better with observed behaviour in different experiments and real-life situations.

## Sammendrag:

Den første delen av dette arbeidsnotatet gir en oversikt over den dominerende innfallsvinkelen innen økonomisk teori for beslutning under usikkerhet — forventet nytteteori. Denne teorien er i økende grad blitt kritisert, og noen av de empiriske brudd med teorien diskuteres. På tross av flere paradokser, og deskriptive og prediktive problemer ved forventet nytteteori, er den fortsatt den dominerende teorien innen tilpasning under usikkerhet. Dette arbeidsnotatet presenterer og diskuterer to alternative ikke-forventet nytteteorier, som bedre kan forklare og predikere observert adferd i ulike eksperimenter og valgsituasjoner i hverdagen.



## Indexing terms:

Risk
Uncertainty
Decision-making theory
Economic theory

## Stikkord:

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Økonomisk teori


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# 1. Introduction[1]

> In the last decade, the economic theory of choice under uncertainty has gone from one of the most settled branches of economics to one of the most unsettled. Although the debate encompasses several topics, it revolves around a single issue: the continued supremacy of the classical "expected utility" model of individual choice under uncertainty, in light of the growing evidence that individuals do not maximize expected utility and the development of a number of alternative "non-expected utility" models of individual decision making (Machina, 1989, page 1622).

The theme of this paper[2] is individual decision making under uncertainty or risk[3], i.e. choice in situations where the consequence of a particular choice is not one, single certain outcome, but rather a set of different outcomes. The focus is on expected utility (EU) theory, where chapter 2 gives a presentation of the theory. Chapter 3 reviews some of the criticism of the EU approach. Some alternative theories, generally known as non-expected utility models, are reviewed in chapter 4.

Table 1 gives a brief account of some of the major developments in the theory of individual choice under uncertainty, and the discussion in this paper will follow this table. One reason for using the EU-theory as our point of departure is that the theory has been the major paradigm in individual decision making under uncertainty since World War II (Schoemaker, 1982). It has had a number of applications to real life situations. 20 years ago it was considered as one of the success stories of economic analysis, with solid axiomatic foundations, important

1 This paper was originally prepared for a PhD course in microeconomics at the Norwegian School of Economics and Business Administration (NHH). It is also part of a study on risk and peasant behaviour and incentives related to natural resource use in developing countries. This paper gives an overview of the standard expected utility theory and some alternative approaches. It does not place the discussion in the context of agricultural decision making in developing countries, which will be done in a later paper. To those interested, a standard reference on agricultural decision making and risk in general is Anderson et al. (1977), whereas Cashdan (1990) and Roumasset (1976) provide good overviews with special reference to developing countries.

2 I would like to thank Kåre P. Hagen of NHH and Ussif Rashid Sumaila at CMI for constructive comments and suggestions that have improved the paper. The responsibility for any remaining errors is, of course, mine.

3 The terms "risk" and "uncertainty" are often used synonymously. The most common distinction, which is due to Frank Knight (1921), is to let risk refer to a situation where alternative outcomes exist with known probabilities, whereas uncertainty refers to a situation where the probabilities are unknown. Knight emphasised the measurability of the probabilities in making the distinction, rather than the extent to which they are known to the decision maker. In most of the more recent literature, this distinction is not made, and the terms are thus used synonymously throughout this paper. In the Knight sense, we are, however, mainly discussing individual choice under risk, not uncertainty.

| *Theory* | *Critique* |
| --- | --- |
| *1.* *Expected value* $EX_i = \Sigma_i p_i x_i$ | St. Petersbourg Paradox |
| *2.* *Expected utility* $EU(X_i) = \Sigma p_i u(x_i)$ | Allais paradox (common consequence effect) |
| *a. Objective probabilities* (Bernoulli, 1738; von Neumann and Morgenstern, 1944) | Common ratio effect<br>Isolation effect<br>Preference reversal<br>Context/framing effect |
| *b. Subjective probabilities* (Savage, 1954) | |
| *3.* *Non-expected utility* | |
| *a. Generalized expected utility* - omitting independence axiom (Machina, 1982 and others) | |
| *b. Regret theory* - omitting transitivity axiom (Loomes and Sugden, 1982) | |

*Table 1: Developments in the theory of individual choice under uncertainty.*

breakthroughs have been made in the analysis of risk, and could provide one of the theoretical building blocks for the newly emerging "information revolution". The situation has changed, and according to Machina (1987), "today choice under uncertainty is a field in flux: the standard theory is challenged on several grounds both within and outside economics" (page 121).

A few notes should be made on issues not touched upon in this paper. First, we shall limit the discussion to single-person decision making problems under exogenous uncertainty. This means we do not enter into, for example, the growing field of principal-agent theory, or issues of market generated uncertainty. Second, we do not explicitly discuss the subjective utility theory, but assume throughout the paper that the decision maker has information on the objective probabilities of the different outcomes. There is a separate school of thought, the post Keynesian perspective (Davidson, 1991), which regards the use of probability theory as of limited usefulness, because most real-world situations are "true" uncertainties, in the Knight sense (see footnote 1).

# 2. The expected utility (EU) theory

## *2.1 Background — the St. Petersburg paradox*

The mathematical form of the expected utility theory goes back to Daniel Bernoulli (1738), who sought to explain the St. Petersburg paradox. This paradox, which was formulated by Daniels cousin Nicholas 10 years earlier, goes as follows:

> You are offered to participate in a gamble, where a fair coin is tossed until it comes up heads. You are paid $1 if it happens on the first toss, $2 if it takes two tosses to land on the head, $4 for three tosses, $8 for four, etc. How much would you be willing to pay to participate in this gamble?

The expected value (EV) of the gamble is:

$$EV = \$1 * (1/2) + \$2 * (1/4) + \$4 * (1/8) + ..... = 1/2 + 1/2 + 1/2 + .... = +\infty$$

The paradox is that people will be willing to pay only a limited amount to enter the game, and below its expected value. The formulation of EV may be unrealistic, but the essence of the argument holds if one agrees to limit the gamble to a finite number of tosses.

The hypothesis put forward by Daniel Bernoulli was that people did not evaluate games according to (and did not maximize) expected net value, but rather expected *utility*. This theory got its more precise formulation by von Neumann and Morgenstern, as seen in the next section.

## *2.2 The von Neumann-Morgenstern axioms*

We assume the following situation for the decision maker (DM): We consider one single good — x, which can be thought of as income or wealth. $x_i$ is the amount of income the DM will receive if state i occurs. The individual chooses between various alternatives or prospects, each of which gives a certain set of probabilities ($p_i$) to each state *i* that occurs. A prospect, P, may be defined as a given income vector, $x = [x_1, ...x_s]$, with an associated probability vector, $p = [p_1, .., p_s]$, with $\Sigma_i p_i = 1$, $i \in [1,s]$:

$$P = (p,x)$$

The entire consequence of any decision is now fully described by a prospect, so the choice between alternative actions is equivalent to the choice between alternative prospects. Different prospects differ in either the probability vector or the income vector, or both. But by including all possible outcomes in a vector x, the differences between the prospects may be more conveniently described only by the differences in the probability vector.

There are different sets of axioms that may produce the expected utility hypothesis. Here we shall present two sets of axiom; first the set associated with the original work of von Neumann and Morgenstern (1944). In fact, the axioms developed by them were slightly different from the ones presented here, which follows the work of Luce and Raiffa (1957). The presentation here is inspired by the textbook presentation in Gravelle and Rees (1992). The second set, referred to as the "modern" one, uses some of the original axioms. The modern set is included because much of the present discussion in the literature refers to this set of axioms. But, first we present the six original axioms of the EU theory.

*Axiom 1: Completeness*

Given a choice between two prospects, $P_1$ and $P_2$, the DM can always state whether the first is preferred to the second ($P_1 \succ P_2$), the second to the first ($P_2 \succ P_1$), or whether he is indifferent between the two ($P_1 \approx P_2$).

*Axiom 2: Transitiveness*

The DMs choice between alternative prospects is transitive. This means that if $P_1 \succ P_2$ and $P_2 \succ P_3$, then we should also have $P_1 \succ P_3$.

*Axiom 3: Preference increasing with probability*

Suppose two alternative prospects, each with only two different states with the same income, $x_u$ (upper) and $x_L$ (lower), i.e. $x_u > x_L$. The probability of $x_u$ is $u_1$ and $u_2$ in the two prospects.

$P_1 = (u_1, x_u, x_L)$
$P_2 = (u_2, x_u, x_L)$

Then the DM would prefer $P_1$ to $P_2$ if and only if $u_1 > u_2$, i.e. if the probability to receive the highest income ($x_u$) is higher with prospect 1. If $u_1 > u_2$, we say that $P_1$ is stochastically dominating $P_2$.

*Axiom 4: Equivalent standard prospects*

Assume we have three different incomes, such that $x_u > x_1 > x_L$. It is then possible to construct a risky alternative, where $x_u$ and $x_L$ are outcomes, such that:

$x_1 \approx P_O = (u_1, x_u, x_L)$

$u_1$ will be unique, i.e. there exists one and only one value of u that makes the DM indifferent between $x_1$ and the risky prospect. $P_O$ is called the *equivalent standard prospect* for $x_1$. Another name for this axiom is the continuity axiom.

*Axiom 5: Rational equivalence*
First we need to define a *compound prospect* as a prospect that for at least one of its outcomes has another prospect, rather than a certain income. To simplify, consider a compound prospect with only two outcomes, where each outcome is a standard prospect:

$$P_c = [p, (u_1,x_u,x_L), (u_2,x_u,x_L)] = (p,P_1,P_2)$$

There are two ways of getting the highest income $x_u$: Either by winning prospect $P_1$ with a probability p, and then get $x_u$ with a probability $u_1$, or by winning $P_2$ with a probability (1-p) and then $x_u$ with a probability $u_2$. Thus, the overall probability for winning $x_u$ is $u_a$:

$$u_a = pu_1 + (1-p)u_2$$

We can now define the *rational equivalent* prospect of $P_c$ as:

$$P_r = (u_a,x_u,x_L)$$

The axiom says that $P_c \approx P_r$. The implications of this axiom is that the decision of the DM is not affected by the two (or more) stage nature of the gamble. He is rational and able to apply the usual method of combining probabilities to arrive at the rational equivalent prospect. There is no risk illusion involved.

*Axiom 6: Context independence*
Consider a set of prospects $P_1,...,P_n$, where $P_j = (p, x_j)$, with p and $x_j$ as vectors of probabilities and state-contingent incomes respectively, and j = 1,..,n. Then, for each state-dependent income, $x_{js}$, we may define an equivalent standard prospect, $P_{js}$:

$$x_{js} \approx P_{js} = (u_{js},x_u,x_L)$$

$x_u$ and $x_L$ will be the highest and lowest income among all $x_{js}$. This determines $u_{js}$ as an increasing function of $x_{js}$. In particular, we note that if $x_{js} = x_u$, then $u_{js} = 1$, and for $x_{js} = x_L$, $u_{js} = 0$. For values of $x_{js}$ between $x_L$ and $x_u$, $u_{js}$ will be between 0 and 1.

$$u_{js} = u(x_{js}) \qquad u'>0$$

If all the state dependent incomes are replaced by the equivalent standard prospect, we obtain n compound prospects:

$$P_{cj} = (p, P_{js})$$

where $P_{js}$ is a vector of all the equivalent standard prospects for the different states (s = 1,...,S).

The sixth axiom states that the DM is indifferent between $P_j$ and $P_{cj}$, i.e. between a given prospect and a compound prospect formed by replacing each income value by its equivalent standard prospect.

The last axiom implies that each of the initial prospects can be transferred to a compound prospect involving only different probabilities of obtaining the same standard prospects. Combining this with the fifth axiom, it means that *any prospect can be written as a standard prospect with only two outcomes, $x_u$ and $x_L$:*

$$P_j = (p, x_j) \approx P_{cj} = (p, P_{js}) \approx P_{Oj} = (u_{aj}, x_u, x_L)$$

According to axiom 3, the problem to the DM is now simply to choose the standard prospect with the highest $u_a$. We can conclude that the DM chooses among the initial prospects as if he maximizes $u_a$. If a particular prospect, $P_k$, is chosen, then we can calculate the corresponding value of $u_{ak}$, which will be larger than all other $u_{js}$ corresponding to the other initial prospects. It is a preference index attached to a probability distribution.

The function u(x) above can be called a utility function, as it gives a representation of the preference ordering on all the prospects. This function is *not* to be interpreted as a quantity of well-being, but simply as a name for the numbers which result when we carry out a series of paired comparisons between prospects.

In the more modern presentations of the EU theory, the set of axioms underlying the theory are somewhat different (see for example Sugden, 1987).[4] There are three essential axioms in this presentation: Ordering, continuity and independence:

*Axiom (i): Ordering*
This is the same axiom as 1 and 2 presented above — the DM should be able to rank all prospects, and the preferences should be consistent in the way required by transitivity.

*Axiom (ii): Continuity*
This is equivalent to axiom 4 above.

4 It may be misleading to call it "modern", as it was in debates, involving, among others, Samuelson and Malinvaud, in the 1950s it became clear that the EU theory rested on these axioms.

*Axiom (iii): Independence*
This axiom is related to axiom 5 and 6 above, and may be formulated in different ways. Consider three prospects, $P_1$, $P_2$ and $P_3$, with $P_1 > P_2$. We define two new prospects, which are a probability mix of $P_1$ and $P_3$, and of $P_2$ and $P_3$, with the same probability (p) of $P_3$ in each of these two prospects. Then, we must have:

$$(p, P_1, P_3) > (p, P_2, P_3)$$

In other words, the prospect containing $P_1$ should be preferred to the one containing $P_2$, for any value of p in the range $0 \leq p \leq 1$. The introduction of a (1-p) probability for winning the third prospect should not influence the ranking. Another name for the independence axiom is independence of irrelevant alternatives, or substitutability (Kreps, 1990),

## 2.3 *The expected utility function*

Given the assumption above, either the "traditional" set of axioms or the "modern" one, the DMs choice will now be as to maximize the expected utility as defined by:

$$Eu = \Sigma_i \, p_i \, u(x_i) \quad i = 1,...,n$$

This is the von-Neumann-Morgenstern utility function. It states that the DM will act as if he maximize a weighted average of the utility in the different states of outcome, where the weights are the probabilities for a state to occur.

Compared to the expected value form ($\Sigma_i \, p_i \, x_i$), the assumption of linearity in payoffs has been dropped. A central property of the expected utility function is that it retains the property of linearities in probabilities. Further, we also note the independence between preferences and probabilities. We will return to these properties in chapter 4.

The utility function is assumed to be at least twice differentiable, with $u' > 0$. The sign of $u''$ depend critically on the preferences towards risk, i.e. whether the DM is a risk lover, risk neutral or risk averter, as discussed below.

Before we proceed, we should also note that the utility function is not unique. Utility is measured on a cardinal scale, but both the origin and the unit of measurement are arbitrary. Thus, any positive linear transformation of the utility function will give the same ranking of probability distributions:

$$v(x) = a + b \, u(x)$$

where $v(x)$ is the new utility function, and a and b constants, $b > 0$. We note that the restriction on valid transformation is stronger than for the standard, ordinal utility function, where any positive monotonic transformation is valid. The reason is that in the case of EU theory the sign of u'' is important, and cannot change during a valid transformation.

## 2.4 *Attitudes towards risk*

To further discuss the attitude towards risk, we first define the *certainty equivalence* of a prospect ($x^{ce}$) as the certain income (p=1) that the DM must get in order to be indifferent to a given prospect P: $x^{ce} \approx P$. In the case with only two outcomes we have:

$$x^{ce} \approx Eu = p\ u(x_1) + (1-p)\ u(x_2)$$

It is customary to distinguish between three different cases, which reflects whether $x^{ce}$ is larger, the same as or smaller than Ex (the expected value of x). The most commonly assumed case is *risk aversion*. The definition is as follows: Assume two prospects with the same expected value, where the first prospect is uncertain and the second gives a certain income. A risk averse DM would prefer the certain income to the uncertain prospect: $x^{ce} < Ex$. The implications on the sign of u'' is derived in the following way:

Assume a prospect with only two outcomes $x_1$ and $x_2$.[5] The DM may receive a certain income equal to the expected value of x: $Ex = px_1 + (1-p)x_2$. Or he may get the prospect, with a probability of p to get $x_1$ and (1-p) to get $x_2$. A risk-averter would prefer the certain income:

$$u(p\ x_1 + (1-p)\ x_2) > p\ u(x_1) + (1-p)\ u(x_2)$$

But this is just the definition of a strictly concave function u(x). Thus, in the case of risk aversion, we have $u'' < 0$. The intuitive interpretation of this result is that because of decreasing marginal utility with respect to income (this is cardinal utility theory), the possibility of a loss of a given size is more important than a gain of the same size. Another way to put it is that a risk averter is unwilling to take a bet which is fair.[6]

[5] The argument can be generalized to more than two outcomes by applying Jensen's inequality.

[6] A fair game is one where the expected (net) value is zero.

We shall only briefly discuss the two other cases of attitudes towards risk, as there are strong empirical evidence that risk aversion is the typical case.[7] If the DM is a risk-lover or gambler, this would imply that $x^{ce} > Ex$, and therefore $u'' > 0$. In the case where the DM is risk neutral, $x^{ce} = Ex$, and $u'' = 0$. In this case, the DM only considers the expected value of the income of the different prospects. In fact, if we assume risk neutrality, we do not need a separate theory for choice under uncertainty, but could simply use the standard theory of individual choice. The outcomes of the different alternatives would be the *expected* values rather than the certain ones, but they would be treated in the same way as certain values.

The existence of risk aversion, and the influence this has on individual decision making, is a major argument for developing a separate theory for individual choice under uncertainty. To apply the theory one needs empirically meaningful measures of the degree of risk aversion. Arrow (1970) provides two concepts or measures of risk aversion, though the first was originally introduced by Pratt (1964). The measures are therefore called the Arrow-Pratt measure of risk aversion:

*Absolute risk aversion:* $R_A (x) = — u''(x) / u'(x)$
*Relative risk aversion:* $R_R (x) = — x\, u''(x) / u'(x)$

One should note that the two measures do not change with a positive linear transformation of the utility function. The relative risk aversion is the same as the elasticity of marginal utility, and therefore also invariant to the units of income. On the size of the two measures, both are positive as $u'' < 0$. Arrow further assumes:

Increasing relative risk aversion with increasing wealth.
Decreasing absolute risk aversion with increasing wealth.

Decreasing $R_A$ implies that a person would be more willing to accept a risk prospect as the income increases. This is supported by intuition and empirical evidence. The case of decreasing $R_R$ is not as obvious, and the mathematical evidence is not unambiguous. Arrow, therefore, concludes that "(1) it is broadly

[7] The existence of insurance markets is the most frequently used argument for risk aversion as the normal risk attitude. However, there are situations which do not confirm to this general assumption. Commonly used examples include gambling and lotteries. Different explanations have been sought to explain this: Friedman and Savage (1948) showed that it may be compatible with the expected utility hypothesis to be risk averse for some types of risk, for example when large amounts are at stake. Further, the introduction of subjective probabilities may also explain this phenomena — people overestimate their chances of winning. Thirdly, the excitement of watching the football match on a Saturday afternoon and see whether one wins or not may be regarded as a good that one buys by taking part in betting (in addition to the gamble itself). Finally, alternative theories have been introduced to explain these phenomena, as we shall come back to in chapter 4.

permissible to assume that relative risk aversion increases with wealth, though theory does not exclude some fluctuations; (2) if, for simplicity, we wish to assume a constant relative risk aversion, the appropriate value is one". For the technical proof of this, see Arrow (1970, pp. 96-98, and 111).

The main tenets of the EU theory can now be summarized as in table 2 (based on Kahneman and Tversky, 1979):

| | |
|---|---|
| *1. EU-theorem:* | Max Eu = $\Sigma_i\, p_i\, u(x_i)$, i = 1,...,n. |
| *2. Asset integration:* | $x_i$ refer to final assets, not gains and losses. |
| *3. Risk aversion:* | $u'' < 0$ (concavity). |

*Table 2. Main tenets of EU-theory.*

## 2.5 *A graphical presentation of the EU model*

The EU model can be presented in an illustrative way in a triangular[8], which would frame much of the later discussion on violations of the EU theory. Consider a prospect with three different outcomes, with $x_1 < x_2 < x_3$, with a corresponding probability vector $p = (p_1,p_2,p_3)$. As $p_1+p_2+p_3 = 1$, we have $p_2 = 1 - p_1 - p_3$. With the three different outcomes given, the utility of that prospect would only depend on $p_1$ and $p_3$.

$$V(p_1,p_3) = p_1\, u(x_1) + (1-p_1-p_3)\, u(x_2) + p_3\, u(x_3)$$ [9]

Now, we define the indifference curve by setting:

$$p_1\, u_1 + (1-p_1-p_3)\, u_2 + p_3\, u_3 = v^*, \text{ or}$$

$$p_1\, (u_1 - u_2) + p_3\, (u_3 - u_2) = v^* - u_2$$

The slope of the indifference curve is given by:

$$(dp_3/dp_1)_{|V=v^*} = (u_2 - u_1)/(u_3 - u_2) > 0$$

8 The origin of this graphical presentation seems to be Marschak (1950) and Machina (1982), and it is therefore often labelled the Marschak-Machina triangle.

9 Following standard notation, we use V(.) to denote the utility index assigned to the whole prospect, and u(.) or $u_i$ to denote the utility of a single outcome.

The curves are shown in figure 1. The direction of increasing preferences is northwest, as this leads to stochastically dominating lotteries; the probability of the higher pay-offs increase as we move north ($x_3$) or west ($x_2$) (cf. the third axiom — preference increasing with probability). Since the equation holds for any value of $v^*$, and $x_1$, $x_2$ and $x_3$ are fixed, it means that the indifference curves will be straight, parallel lines in the ($p_1$,$p_3$) diagram. Using the "modern" EU axioms, the existence of indifference curves is a result of the ordering and continuity axioms, whereas the property of parallel lines follows from the independence axiom.

Similarly, we can construct the iso-expected value lines, defined as:

$$Ex = p_1\ x_1 + p_2\ x_2 + p_3\ x_3 = p_1\ x_1 + (1 - p_1 - p_3)x_2 + p_3\ x_3 = x^* \text{ (constant)}$$

The slope of this curve is given by:

$$(dp_3/dp_1)_{|Ex=x^*} = (x_2 - x_1)/(x_3 - x_2) > 0$$

These curves will also obviously be straight, parallel lines. Movements northeast along an iso-expected value line in the ($p_1$,$p_3$)-diagram implies that the probability of $x_2$ ($p_2$) decreases, while $p_1$ and $p_3$ increase. In other words, the chances for the middle income ($x_2$) decreases, whereas the probability of the tail outcomes increases. Thus, we have an example of mean preserving spreads, or pure increases in risk (as defined by Rothschild and Stiglitz, 1970).

What would the slope of the iso-expected value curve be compared to the indifference curve? If the DM is risk averse, i.e. u(.) is concave, we have by definition when $x_1 < x_2 < x_3$:

$$(u_2 - u_1)/(x_2 - x_1) > (u_3 - u_2)/(x_3 - x_2)$$

Rewriting this we get:

$$(u_2 - u_1)/(u_3 - u_2) > (x_2 - x_1)/(x_3 - x_2)$$

This inequality says that the indifference curves are steeper than the iso-expected value curves when the DM is a risk averter. The steeper the indifference curves, the more risk averse the DM. Similarly, if the DM is a risk lover, the indifference curves would be more flat than the iso-expected value curves, whereas the curves would be identical in the case of risk neutrality.

From the diagram, we see clearly that a risk averter would prefer any southwest movements along an iso-expected value line. This would reduce the risk, while the expected value of the gamble is maintained.

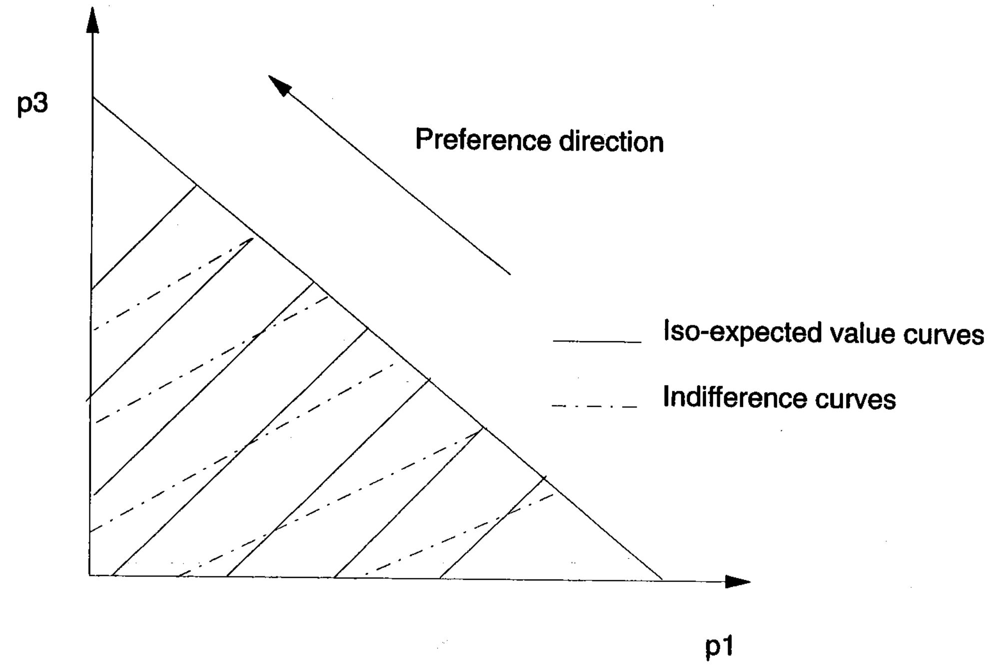


*Fig. 1: Expected utility theory in a ($p_1$,$p_3$)-diagram or Marschak-Machina triangle.*

If we observe the preferences around a prospect (that is testing the choice between prospects with slightly different probability vectors), we may construct a local utility function. Moreover, because the indifference curves are straight, parallel lines, we know that the slope of all local utility function is the same, and we have therefore found the global utility function. Thus, the utility function can be constructed by just observing the preferences locally.

## 3. Critique of the expected utility (EU) theory

### *3.1 Alternative interpretations of the EU theory*

The EU theory can be interpreted in different ways, based on what is considered the purpose of the model. Thus, in order to access the acceptability of a model, one first has to clarify what should be the basic objective of the model. Schoemaker (1982) distinguish between four different purposes the EU model can serve:

*1. Descriptive model*

This view emphasises the explanatory aspect of the theory, and the importance of the realism about the assumptions themselves. The model should describe the way decisions are taken, including the manner in which information is processed by the DM.

*2. Predictive or positivistic model*

Realism about the axioms and postulated computational mechanisms are not important according to this interpretation; what matters is the accuracy of predictions (compared to competing models). This view is advocated by Milton Friedman and others.[10] If a theory is able to produce predictions that are in line with actual behaviour, it does not matter whether the assumptions are right or wrong. "The proof of the pudding lies in the eating" — and not the recipe. An additional argument in the case of the EU theory is that the theory can be developed from different sets of assumptions, of which only one is presented here.

The border line between the descriptive and the predictive views is not sharp. First, one can argue that a theory with realistic assumptions are more likely to produce good predictions. Second, the assumptions or axioms themselves are hypothesis or predictions about the DM's behaviour.

*3. Postdictive model*

"The essential premise of the postdictive EU view is that all *observed* human behaviour is optimal (in the EU sense), *provided* it is modeled in the appropriate manner. Seeming suboptimalities are explained, ex post facto, by introducing new considerations (e.g. costs, dimensions, constraints, etc.) that account for the anomalies, so as to make them optimal" (Shoemaker, 1982, page 539). Any violations of the EU theory would, under this interpretation be illusory, because it may be argued that it is due to improper specification of the model. Under this interpretation, the optimality of economic behaviour is a meta-postulate, and any unexplained behaviour by a model is due to the fact that the model is too simple. Stigler and Becker (1977) argue that many phenomena that are often interpreted as departures from optimal economic behaviour is actually optimal if the models are expanded slightly. Others would regret this postdictive perspective in economics, because it has the danger of tautology or circularity: Whatever an economic agent chooses is optimal, and he chooses what he does because it is optimal. Particularly when economics is expanded into new domains, as for example Gary Becker (1976) does, the tautological nature of models may increase. (See Scitovsky, 1976, for an elaboration of this view).

Another key concept worth mentioning in this connection is Simons (1955) "bounded rationality". It is argued that individuals are rational, but not in the simple way normally assumed in economics. There are important limitations in the computational and information-processing capabilities, which forms a constraint on the DMs choice. Considering the costs of, for example, obtaining more information, the optimal choice may be to be approximately optimal (using "optimal" in the first case as a more general type of optimization, including all

[10] See Friedman (1953).

relevant aspects, and in the latter case in the way it is defined in the standard EU theory).

*4. Prescriptive or normative model*
In the normative interpretation, the EU theory tells us how an economic actor ought to behave if he is rational (in the way defined by the axioms) and wants to maximize his own utility. Thus the purpose of the model is prescriptive, i.e. to improve the decisions.

The main criticism of the EU model is both as a descriptive and predictive model. It is argued that the EU theory is of limited use in understanding, explaining, and predicting how economic agents make decisions in situations of uncertainty. A large part of the criticism have been based on "laboratory" experiments, where one creates gambles to see whether a selected group of people make choices that are consistent with the EU theory. Experiments in real-life situations have also been conducted, extending from financial markets to Las Vegas casinos. Even the use of laboratory rats, choosing among gambles which involve variations in daily food, have been tried to test the universality of the EU theory.[11]

Particular attention has been given to the independence axiom, which may be considered the strongest of the three axioms in the "modern" set of axioms. The ordering and continuity axiom are parallel to the axioms used in the standard theory of individual choice under certainty (consumer theory), and generally considered more acceptable. We shall in the following sections review some of these criticisms. The literature on the empirical testing of the EU theory is very large, and only a very tiny fraction is touched upon here, and related to some alternative theories. Excellent surveys are provided in Schoemaker (1982), Machina (1987), and Appleby and Starmer (1987).

## *3.2 The Allais paradox or common consequence effect*

The Allais (1953) paradox is the best known example of systematic violations of linearity in probabilities or the independence axiom. The paradox goes as follows: You have to select between two pairs of gambles; in the first you choose between $P_1$ and $P_2$, in the second between $P_3$ and $P_4$, defined as follows:

$P_1 = (1, 0.5m)$, i.e. a sure income of \$ 500 000
$P_2 = ([0.1, 0.89, 0.01], [2.5m, 0.5m, 0])$

$P_3 = ([0.11, 0.89], [0.5m, 0])$
$P_4 = ([0.1, 0.9], [2.5m, 0])$

[11] See Battalio, Kagel and MacDonald (1985).

Allais found that the majority of people would prefer $P_1$ to $P_2$, and $P_4$ to $P_3$, which at a glance seems to be quite reasonable. In a similar formulation, Kahneman and Tversky (1979) found that 82 pct. chose $P_1$ and only 17 pct. $P_3$.[12] But these preferences are *not* consistent with the EU theory: $P_1 > P_2$ implies:

$$u(0.5m) > 0.1\ u(2.5m) + 0.89\ u(0.5m) + 0.01\ u(0), \text{ or}$$

$$u(0.5m) - 0.89\ u(0.5m) = 0.11\ u(0.5m) > 0.1\ u(2.5m) + 0.01\ u(0)$$

$P_4 > P_3$ implies:

$$0.11\ u(0.5m) + 0.89\ u(0) < 0.1\ u(2.5m) + 0.9\ u(0), \text{ or}$$

$$0.11\ u(0.5m) < 0.1\ u(2.5m) + 0.01\ u(0),$$

which is the opposite of what we found to be the implication of $P_1 > P_2$. Thus, what seemed to be a reasonable choice between the two pairs of prospects, and what most people actually would choose, is contradicting the logic of the EU theory.

The Allais paradox or the common consequence effect[13] can also be illustrated using a $(p_1,p_3)$ diagram as done earlier. We note that the different prospects are variations in the probabilities of winning the three outcomes (0, 0.5m, 2.5m), where the corresponding probability vectors are:

$p_1 = (0, 1, 0)$
$p_2 = (0.01, 0.89, 0.1)$
$p_3 = (0.89, 0.11, 0)$
$p_4 = (0.9, 0, 0.1)$

Particularly, we note in the example that the lines between $P_1$ and $P_2$, and between $P_3$ and $P_4$ are parallel. As the indifference curves also are parallel lines, we see that consistency according to the EU theory implies that the DM choose either $P_1$ and $P_3$, or $P_2$ and $P_4$.

12 Leonard J. Savage, one of the major contributors to the EU theory, had this type of preferences when first confronted with this example, but concluded upon reflection that there were some errors in his preferences (Savage, 1954). He is an exception, however, as most people do not change their initial preferences even after the inconsistencies in their choices have been explained. They rather start arguing with the assumptions of the EUA. See Schoemaker (1982, page 555) for a brief review.

13 This name of the phenomena is due to the common consequences or outcomes of 0.5 in $P_1$ and $P_2$, and 0 in $P_3$ and $P_4$.

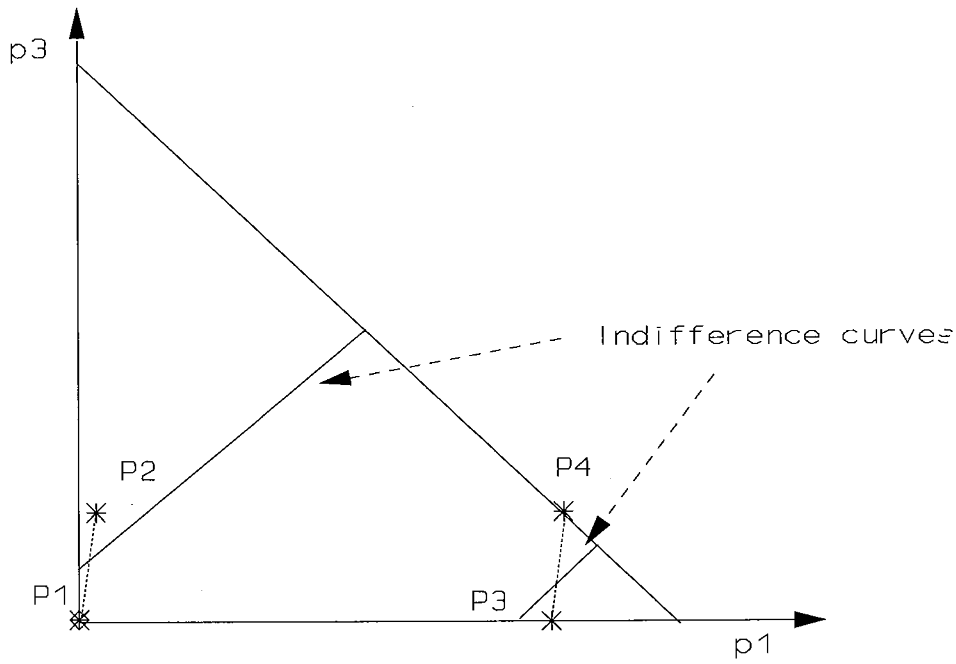


*Fig. 2: The Allais paradox or common consequence effect*

We note that a set of indifference curves that would be consistent with the behaviour observed must be less steep in the southeast area than in the middle and northwest part of the triangle.

### *3.3 The common ratio (or certainty) effect*

We shall now consider a second test of the EU model, known as the common ratio effect (Kahneman and Tversky, 1979). The subjects are offered two pair of prospects:

$P_1$ = (1, 3 000)
$P_2$ = ([0.8, 0.2], [4 000, 0])

$P_3$ = ([0.25, 0.75], [3 000, 0])
$P_4$ = ([0.2, 0.8], [4 000, 0])

In their experiment, 80 pct. of the subjects chose $P_1$ in the first pair, whereas only 35 pct. preferred $P_3$ in the second pair. EU theory implies, as can be shown in the same way as above, that if $P_1 > P_2$, then $P_3 > P_4$, and vice versa. This violation of the EU model is known as the common ratio effect.[14]

The common consequence effect and the common ratio effect seems to be closely related to each other. In both cases, people are attracted to a certain gain ($P_1$) rather than a gamble with a slightly higher expected value ($P_2$). But when it comes to games where the chance of winning anything at all is relatively small, people seem attracted to the gamble with the largest prize ($P_4$).

This intuition does not, however, hold for a version of the common ratio effect, which was also tested by Kahneman and Tversky. In this prospect we just replace the gains in the gamble above with a loss of the same amount. The prospects in this experiment would then be as follows:

$P_1$ = (1, -3 000)
$P_2$ = ([0.8, 0.2], [-4 000, 0])

$P_3$ = ([0.25, 0.75], [-3 000, 0])
$P_4$ = ([0.2, 0.8], [-4 000, 0])

Again, EU theory implies $P_1 > P_2$ and $P_3 > P_4$; or $P_1 < P_2$ and $P_3 < P_4$; or $P_1 \approx P_2$ and $P_3 \approx P_4$. Conducting the experiment, they found that 92 pct. chose $P_2$, and 42 pct. $P_4$. What is interesting is that people choose a truly unfair gamble, i.e. $P_2$ ($Ex_2$ = -3 200), in preference to a certain one ($Ex_1$ = -3 000). When it comes to the second choice, where the chances of loosing anything is relatively small, the subjects were attracted by the gamble whose worse outcome is less bad, i.e. $P_3$.

Other examples of similar violations of the independence axiom includes the "Bergen paradox" by Hagen (1979), which is a special case of the common ratio effect.

A third category of violations of the independence axiom is the "isolation effect" in two-stage gambles. This is not discussed further here, see Kahneman and Tversky (1979) for the details of this paradox.

### *3.4 Preference reversal phenomena*

This phenomena, which was first reported by the psychologists Lichtenstein and Slovic (1971), is another example of violations of the EU model. According to the EU theory, the DM should choose the prospect with the highest certainty equivalent. This property is also assumed in the non-expected utility model of

[14] The name common ratio effect comes from the common ratio between the probabilities in the two pair of prospects, i.e. 1/0.8 = 0.25/0.2.

Chew-Fishburn discussed in chapter 4.1. However, in this experiment (see for example Machina, 1987 for the details), a systematic tendency for violation of this prediction was found: People chose prospect A in favour of B, even though they assigned a higher certainty equivalent to B when asked to value the two prospects. A possible explanation as to why people chose in the way they did is "that choices among pair of gambles appeared to be influenced primarily by probabilities of winning and loosing, whereas buying and selling prices were primarily determined by the dollar amount that could be won or lost" (Slovic, Fischoff and Lichtenstein, 1983). Thus, this is a case of intransitivity, and may be used as an argument for the regret theory discussed in chapter 4.2.

### *3.5 Context or framing effects*

Several studies have shown the importance of context of framing of the gamble. This may significantly affect the actual choices, even though the underlying prospects are the same. Hershey and Schoemaker (1980) illustrates this by presenting the following pair of prospects to a group of people:

*Gamble formulation:*
$P_1$: A sure loss of $10.
$P_2$: A 0.01 chance of loosing $1 000

*Insurance formulation:*
$P_3$: Pay an insurance premium of $10.
$P_4$: Remain exposed to a hazard of loosing $1 000 with a 0.01 chance.

According to EU theory, the gamble and insurance formulation has identical underlying prospects, but they turn out to be quite different psychologically. 56 pct. preferred $P_1$ and 81 pct. preferred $P_3$. Hershey and Schoemaker further found that this discrepancy was strongest for probability and loss levels representative to insurance hazards, i.e. low probabilities and large loss. Further, the insurance formulation evoked greater risk aversion than the gamble formulation. Possible explanations on this phenomena is that the different formulations evoke different social norms, or that the insurance formulation gives the impression that something is gained. The problem of context effects is real, but may also be present in other approaches than EU theory.

## 4. Alternative approaches to decision making under uncertainty

### *4.1 The fanning out hypothesis and non-expected utility models*

The above examples are violations of the assumption of linearity in probabilities (produced by the independence axiom). It has been shown that all of these violations can be characterized in the same way. This has generated the *fanning out hypothesis*, i.e. the indifference curves have a pattern like a fan, where the curves are steeper as we move northwest in the diagram. This was the case of the Allais paradox in figure 2. The theory discussed in the following is due to Machina (1982). More easily accessible presentations are found in Machina (1987) and Sugden (1987); this presentation is based on the latter. This formulation is part of what is more generally referred to as *non-expected utility theory*. The Machina model is also being referred to as the generalized expected utility theories.[15]

To describe the characteristics of individual behaviour in situations of uncertainty, we need a non-linear utility function, i.e. one which is not linear in the probabilities. Machina (1982) discusses the general properties such a nonlinear utility function — H(x) — should have, where the two most important are:

(1) H(x) should be increasing in x, which ensures that stochastically dominating distributions are always preferred.[16] In the $(p_1$-$p_3)$-diagram, it means that the indifference curves should slope upwards.

(2) Let R(x;F) be the Arrow-Pratt measure of absolute risk aversion, where F is a probability distribution:

$$R(x;F) = - (\delta^2 H(x;F)/\delta x^2) \ / \ (\delta H(x;F)/\delta x)$$

For any two distributions, where $F_1$ stochastically dominates $F_2$, we have $R(x;F_1) \geq R(x;F_2)$. This means that as we move towards stochastically dominating distributions, the degree of (local) risk aversion associated with any given level of wealth is non-decreasing. In the diagram, this implies that as we move north or west (stochastically dominating distributions), the indifference curves will be steeper (more risk averse). These properties will produce the fanning out of the indifference curves.

A number of mathematical forms of the non-linear utility function has been suggested in the literature, see Machina (1987) for some of these. A simple form, which has proved to be very useful both theoretically and empirically, has been suggested by Chew (1983) and Fishburn (1983):

$$V(p_1,..p_n) = \Sigma_i \, p_i \, u(x_i) \ / \ \Sigma_i \, p_i \, w(x_i)$$

[15] One branch of the non-expected utility theories is the prospect theory (Kahneman and Tversky, 1979), which is not discussed in this paper.

[16] Stochastic dominance (of first order) is defined as follows: Let Z be stochastic income, and F and G two cumulative distribution functions. F is stochastically dominating G if $G(Z) \geq F(Z)$ for all $Z \in [Z_1, Z_2]$,

u(.) and w(.) are two different utility functions, of the same kind used in the standard EU theory. In the special case where $w(x_i) = 1$, we are back to the standard EU theory.

An indifference curve in the $(p_1,p_3)$-diagram is defined by $V(p_1,p_2,p_3) = v^*$. With the new functional form, we get:

$$[p_1 u_1 + (1-p_1-p_3) u_2 + p_3 u_3] / [p_1 w_1 + (1-p_1-p_3) w_2 + p_3 w_3] = v^*$$

Rearranging this expression, we obtain the following expression:

$$p_1 [-(u_1 - u_2) + v^*(w_1 - w_2)] + p_3 [-(u_3 - u_2) + v^*(w_3 - w_2)] = u_2 - v^*w_2$$

The slope of the indifference curve is given by:

$$(dp_3/dp_1)_{|V=v^*} = - [-(u_1 - u_2) + v^*(w_1 - w_2)] / [-(u_3 - u_2) + v^*(w_3 - w_2)] > 0$$

All variables at the right hand side are constants, so the indifference curves are linear. Further, it can be shown that only two such equations for the indifference curve, say $V(p_1,p_2,p_3) = v^*$ and $V(p_1,p_2,p_3) = v^{**}$, can be linearly independent. Thus, the equation for the indifference curve defines a set of lines which all intersect at the same point. To be meaningful, this point must lie outside the triangle of meaningful prospects (i.e. where $0 \leq p_1 + p_3 \leq 1$), as illustrated in figure 3. The intersection point is located southwest of the origin.

The intuition behind the fanning out hypothesis is that people systematically overestimate low probability outcomes. Thus, it is consistent with the general observations on insurance and lotteries, i.e. that people in some situations prefer to insure themselves against risk (even though the expected value may be negative due to transaction costs), but in other situations participate in unfair lotteries (i.e. where the expected value of net gain is negative). Why this is consistent with this theory is clearly seen from figure 3. A *lottery* would have a small chance of winning the high outcome ($p_3$), thus we are in the south-east part of the triangle. Here the indifference curves are relatively flat, and less steep than the iso-expected value curves. The DM behaves as a risk-lover and may therefore be willing to accept an unfair lottery. Similarly, an *insurance* situation would be located in the northwest area of the triangle, where the prospects of loosing are relatively small ($p_1$), but the consequences of state 1 very bad for the DM.[17] The indifference curves are relatively steep in this region (steeper than the iso-expected value

[17] One should notice that we are not in the same triangle for the lottery and insurance situations, since the outcomes $(x_1,x_2,x_3)$ have changed.

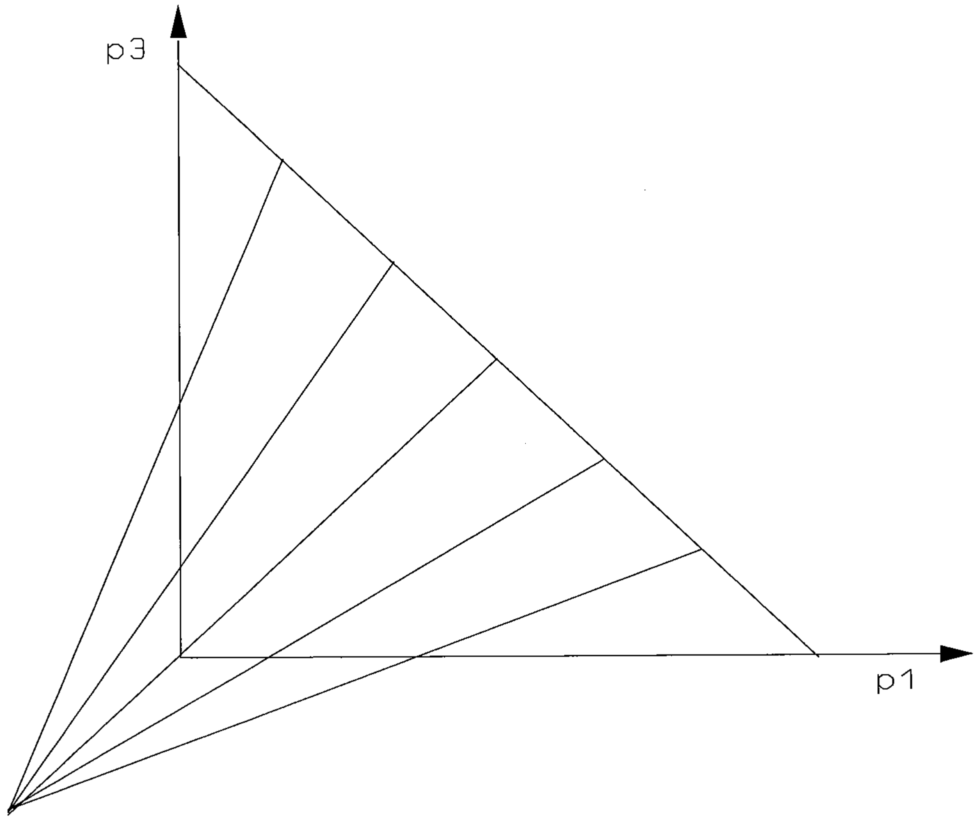


*Fig. 3: The fanning out hypothesis in a $(p_1, p_3)$-diagram.*

curves). This is equivalent to a high degree of risk aversion.[18]

## *4.2 Regret theory*

The criticism of the independence axiom discussed above has been based on the interpretations of the EU theory as a descriptive and predictive model. How should these violations of the axiom be interpreted when we use the standard EU model as a prescriptive or normative model? As Sugden (1987) suggests, "we seem forced to conclude *either* that ordinary people are irrational *or* that the independence axiom is not, after all, a necessary property of rational choice" (page 15). He uses an analogy to consumer theory. Consider three different bundles of physical goods; p, q and r. It does *not* follow that if p is preferred to q, then a 50-50 mix of p and r should be preferred to q and r, because there may be complementarities between the goods. According to Samuelson (1952), the key

[18] We should note that it is more problematic to use the term risk aversion now, because the slope of the indifference curves is not only determined by u(.) as in the standard EU theory, but also the decision weights used, i.e. the subjective weights given to the objective probabilities.

difference between the physical commodity mixtures and prospects of probability mixtures, is that the former may be consumed simultaneously, whereas the prospects in a probability mixture are mutually exclusive. However, the fact that people violate the independence axiom suggests some complementary effects also in the case of mutually exclusive prospects. This is at the psychological level, where for example the disappointment of ending up with an outcome worse than expected, affects the choice.

Then, we end up with the question of whether disappointment should be part of a theory of rational choice or not. Loomes and Sugden (1985) suggests that it may, and they have (among others) developed the *regret theory*. We shall only briefly review this theory here.

The basis for the regret theory is that "the utility which you derive from a particular consequence of one action may be influenced by a consequence of a *different* action" (Sugden, 1987, page 16). Let $x_i$ be the outcome one gets from the actual choice taken, and $x_j$ be the outcome *if* one had chosen a different action. If $x_i > x_j$, there is some rejoicing involved because one did chose right given that this particular state occurred. Similarly there is a regret if it turns out that $x_i < x_j$. Suppose there are only two prospects, each with a probability vector p and q, defines over a vector of consequences x. The prospects are further assumed to be statistically independent. The utility is indexed according to a modified utility function, which takes account of regret or rejoicing: $M(x_i, x_j)$. The chance of getting $x_i$ and miss $x_j$ is $p_i q_j$, so the DM would maximize:

$$\Sigma_i \Sigma_j p_i q_j \, M(x_i, x_j)$$

A possible form of M(.) suggested by Loomes and Sugden (1982) is

$$M(x_i, x_j) = C(x_i) + R[C(x_i) - C(x_j)]$$

where C(.) is the "basic" utility function, and R(.), R' > 0, is a regret-rejoice function that assigns an increment or decrement of utility, depending on the difference of "what is" and "what might have been".

We shall not go further into the technical details of this theory. What is surprising, is the convergence between the regret theory and the Chew-Fishburn formulation of the fanning out hypothesis given above. The transitivity or non-transitivity of preferences is the only real difference between the regret theory and the Chew-Fishburn formulation of the non-expected utility theory. It means that if we drop the transitivity axiom (which is a part of the ordering axiom in the modern version), what we get is the regret theory. For a proof of this result, see Sugden (1987, pages 18-22).

## 5. Concluding remarks

The success of the EU model is in part due to its simple and general form, and the bold and testable predictions produced by the model. As this paper has briefly reviewed, there is strong evidence of systematic violations of the theory. Schoemaker (1982) concludes, after having reviewed a large number of empirical studies, "that at the individual level EU maximization is more the exception than the rule" (page 552). He summarizes the failure of EU theory in three points: First, people do not structure problems in a holistic way, i.e. they do not evaluate one alternative independently of the other alternatives in the choice set (cf. regret theory). Second, people do not process information, and particularly information on probabilities, according to the EU theory. Specifically, there is a tendency of overestimation (underestimation) of desired (undesired) outcomes, and low probabilities are given higher "decision weights" than their objective value. Third, the EU theory poorly predicts actual behaviour in laboratory situations. He notes, however, that there may be exceptions: "For well-structured repetitive tasks, with important stakes, and well trained decision makers, EU maximization may well describe the actual decision process, e.g. oil drilling decisions" (page 552).

Any modifications of the theory will increase the complexity, and we have the classical trade-off between simplicity, and the quality of the theory as a descriptive and predictive one. Moreover, the evidence from empirical testing of the EU theory is not uniform, and a theory compatible with all the evidence is yet to be developed. The large body of evidence that do not confirm the EU model may, however, suggest that alternative theories, which seems to be *more* consistent with actually observed behaviour, should get a more prominent place in standard presentations of the economic theory of individual choice under uncertainty. The fact that standard graduate microeconomic textbooks like Gravelle and Rees (1992), Kreps (1990) and Varian (1992) do not include such theories may indicate that there is some disagreement about the value of the non-expected utility theories within the discipline.

Machina (1989) discusses three goals which the non-expected utility theory has to meet for it to become adopted by economists; an empirical, a theoretical and a normative goal. The empirical goal is to demonstrate that non-expected utility models fit data better than the standard EU model. The theoretical goal is to show that the new models can be used to conduct analysis of standard economic decisions in situations of uncertainty. Machina claims that the non-expected utility models have been particularly successful in meeting the empirical goal, and are increasingly meeting the theoretical goal too. Regarding the normative goal, that is whether non-expected utility models should be used for normative purposes, there is a strong hesitation within mainstream economics to accept departures from the standard EU model. It is generally claimed that individuals that are non-expected utility maximizers may be subject to systematic manipulation and exploitation. For example, the regret theory does not make use of the transitivity axiom. An individual with intransitive preferences "will get eaten alive by a simple "money pump" argument" (page 1623). Similar arguments can be raised against omitting the independence axiom, though Machina (1989) raises doubts about the validity of these arguments. He tries to make a compromise, by suggesting that the normative acceptability of the independence axiom (or separability across mutually exclusive events) may depend on the level of consequence description. At the level of description normally used by economists (monetary outcomes), there are strong normative reasons why preferences may be nonseparable. At a deeper level, where any relevant emotional state is included, separability may be rational.

Still, we may continue to see some disagreement between those who view economic analysis as the description and prediction of what is considered rational economic behaviour (in the EU sense), and those who view it as the description and prediction of observed behaviour. The hesitation to accept models which describe human behaviour that may be considered irrational from the standard EU model seems to be strong within mainstream economics. But the signs of increasing pluralism in the theories of individual choice under uncertainty indicate that the discipline, for some time, may have several competing approaches coexisting.

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