Shifting Cultivation Expansion and Intensity of Production: The Open Economy Case

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Summary:

This paper studies decision making in shifting cultivation, in particular labour inputs, length of rotation or fallow period (intensity of production), and the agricultural frontier (expansion). Analytical models are developed, combining forest rotation and spatial approaches in resource economics. The small, open economy assumption is used, that is, all prices, including the wage rate, are fixed in the models. This is crucial for the effects of various policies. Three different property rights regimes are discussed: Social planner's solution with secure rights to all forestland, open access, and homesteading, where property rights are established through forest clearance.

Sammendrag:

Dette arbeidsnotatet analyserer beslutninger i svedjebruk, spesielt arbeidsinnsats, rotasjonslengde og ekspansjon. Analytiske modeller utvikles ved bruk av to ulike innfallsvinkler i ressursøkonomi for skogsrotasjon og lokalisering. En bruker forutsetningen om en liten, åpen økonomi, dvs. at alle priser, inkludert lønn, er gitt i modellene. Dette er en kritisk forutsetning for effekten av ulike typer politikk. Tre ulike regimer for eiendomsretter diskuteres: En sentral samfunnsplanlegger med sikre rettigheter til all skog, en situasjon med fri adgang ("open access"), og en situasjon hvor nyrydding av skog gir bonden eiendomsrettigheter.

Indexing terms:

Shifting cultivation
Agriculture
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Stikkord:

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1 Introduction and overview[1]

1.1 The importance of shifting cultivation

Shifting cultivation is an agricultural practice at an early stage in the evolution of agricultural systems (Boserup, 1965; Ruthenberg, 1980). The system is characterized by abundant land, whereas labour is considered the constraining factor. A system with effortless self-fertilization of the soil through a long fallow period, and burning of the vegetation before one or a few years of cropping, is therefore a rational response from the farmers side to the relative scarcity of inputs. In this situation, the shifting cultivation system may yield higher output per unit labour input than sedentary systems (Boserup, 1965; World Bank, 1990).

Most studies of shifting cultivation are either within the anthropological sphere, typically focusing on how the production system is integrated in a wider cultural and social structure, or the soil science sphere, focusing on issues like erosion and nutrient cycling (see Robinson and McKean, 1992, for an extensive bibliography). There are relatively few *economic* studies and models of shifting cultivators' behaviour and decision-making. Exceptions include Dvorak (1992), who develops a simple model with no costs of expanding land use and a subsistence requirement; Holden (1993) uses a Chayanov (1966) approach and develops linear programming models to study of shifting cultivation in northern Zambia; Nghiep (1986) similarly uses a LP model to study conditions for agricultural transformation in Brazil; whereas López and Niklitschek (1991) develop a more general dual (two sector) model. This paper presents an alternative economic approach to the study of shifting cultivation, which focuses on two particular characteristics of the shifting cultivation system, that is its *spatial* dimension, and the *forest rotation* aspect. These issues are not treated satisfactory in the above models.

There is a number of good reasons as to why shifting cultivation deserves further economic analysis and modelling. The deficiency of economics modelling provides an argument in itself, as the nature of economic decision-making and farmers' response to exogenous changes need to be better understood in order to design effective policy instruments. Policy makers may want to influence the development of shifting cultivation for both environmental, social, economic, and political reasons. The problems of deforestation and soil erosion related to expansion of shifting cultivation are well established. Shifting cultivation is commonly being held responsible for about half of tropical deforestation (see, however, Angelsen, 1994, for a critical discussion of this estimate, and of the different environmental effects of various forestland uses). Some governments focus on the extensive nature of the practice, considering it an inefficient use of forestland (high opportunity costs).

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[1] I would like to thank Röngvaldur Hanneson, Stein Holden, Karl Pedersen, Ussif Rashid Sumaila, and Arne Wiig for comments on draft versions of this paper. Remaining errors and omissions are my responsibility.

On the political side, governments may also want to "develop" shifting cultivation into more permanent settlements which may be easier to control politically, or due to the economics of scale in the supply of public services. Others argue that the "primitive" nature of shifting cultivation may not correspond to the image of progress that governments want to present (Dove, 1983). Whereas our sympathy for such arguments are limited, the fact that there exist important negative external (environmental) effects, provides a sufficient rationale for the study of shifting cultivation.

Moreover, the often low incomes among shifting cultivators make increasing agricultural income an important element in the combat against poverty. The key challenge is how to enhance the output from the system, while maintaining its long term productivity (e.g., soil erosion), and avoid losses in other environmental functions (e.g., expansion into virgin forest which reduce the biodiversity). In other words, how to achieve a *sustainable intensification* of the system. There is no easy answer to this end, and it may even entail important trade-offs in some situations: The concern for the system's long term productivity indicates longer fallow periods, whereas the goal of limiting its expansion may call for an intensification through shorter fallow periods. Thus, we may have conflicts between short and long term productivity, and between production and environmental conservation objectives.[2]

This paper will focus on how three key variables in the shifting cultivation production system are determined and affected by changes in various exogenous parameters. The endogenous variables are (1) the agricultural frontier or maximum distance of cultivation from a village centre, which then determine total agricultural land and deforestation; (2) the length of the fallow period (that is, the inverse of the intensity of production); and (3) labour inputs.

1.2 The von Thünen approach

The models in this paper make use of and integrate two different approaches in agricultural and resource economics: *Spatial* models in the von Thünen (1826) tradition, and *forest rotation* models in the Faustman (1849) tradition.

In the von Thünen models transport costs and accessibility play a crucial role in determining the land rent and the agricultural frontier, and thereby land area under cultivation. In this approach, land is assumed to be homogenous, and differs only by the location as measured by distance from a centre (village). This is contrasted with the Ricardo approach, where distance costs are neglected, but land differs in quality (soil fertility). Including differences in fertility would add another dimension to the problem, but not change any of the main results presented in this paper (see Randall and Castle, 1985, for a comparison).

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[2] This is elaborated in Angelsen (1993).

In the von Thünen model land is assumed to be physically infinite.[3] There is, however, scarcity of *good* land, that is land close to the centre (land with low distance costs). The land frontier or the border between cultivation and virgin forest will be determined endogenously. A basic premise in the model here is that all forestland which yield a positive land rent will be converted to agricultural use. All land rent will be captured. The frontier is defined as where the rent is zero.

A large body of studies in the von Thünen tradition focuses on how different activities are located in zones of different distance from the centre, depending on their transport costs, e.g., value/weight ratio for agricultural products (Randall and Castle, 1985). We ignore this aspect, and consider only the choice between one activity, that is shifting cultivation, and virgin forest. Further, we only deal with one homogenous agricultural crop, and do not discuss the choice between different crops, in particular between annuals and perennials. These choices could be solved by using the usual "brute force" methods, i.e., to compare the maximum value of the objective function for different land uses or crops. Implicitly, we assume that these choices already have been made, and we consider the most profitable (mix of) agricultural product.

This paper is within the branch of spatial models which takes the centre as given, and a transport network already in place, i.e., a *partial equilibrium approach* (Starret, 1974). Thus we do not address some important issues, including endogenous changes in the transport system, formation of new villages or centres, and expansion of existing ones. This could be an acceptable simplification if the costs of establishing new centres are very high, and the transport system is a result of exogenous decisions. The latter is clearly the case in my study area in Seberida, Sumatra as well as many other areas in Southeast Asia, where road construction and other infrastructure developments have been closely connected to government sponsored projects like large-scale logging, plantations, and transmigration.

1.3 The Faustman approach

In the Faustman (1849) forest rotation models the optimal age of the forest at the time of cutting is discussed under various assumptions (discount rate, relative prices, costs, technology, risk, environmental effects, etc.). Most models developed in this tradition, like the one presented in this paper, assume all important parameters to be constant over time, and then discuss changes in the steady-state from one-time changes in exogenous variables. Thus, the models deal with different long-term bio-economic equilibria; there is, for example, no land degradation over time (the production function remains constant). The model does

3 Or in the words of von Thünen himself: "Imagine a very large city in the midst of a fertile plain not traversed by any navigable river. The plain's soil is of uniform quality and capable of cultivation everywhere. At a great distance from the city the plain turns into an uncultivated wilderness separating this state from the rest of the world. The question is this: under these conditions what kind of agriculture will develop and how will the distance to the city affect the use of land if this is chosen with the utmost rationality?" (Quoted in Beckman, 1972: 1.)

not either deal with possible irreversibilities involved. These are crucial assumptions, which simplify the analytics tremendously, as a dynamic problem is reduced to a static optimization problem. To include land degradation over time calls for more truly dynamic methods like dynamic programming.

The obvious similarity between timber production and shifting cultivation is the *rotation* aspect and the cyclical harvesting of a renewable resource. However, applying models of timber production to shifting cultivation requires several modifications. First, the benefits and costs involved are different, e.g., costs of planting trees are normally not present in shifting cultivation, whereas the clearing of forest is the start of a production cycle that involves labour inputs for planting, weeding, pest control, harvesting, etc. This paper explores how forest rotation models could be reformulated to the shifting cultivation setting.

More important, timber economics models normally assume private or government operated forests with well defined and secure property rights, and competitive input (including labour) and output markets. This may not always be the case in a shifting cultivation setting. This and a companion paper intend to carry out a *structural sensitivity analysis,* that is to see how the outcome and effect of various policies depend on the economic structures, here defined as different assumptions about the labour market and the property regime (see below).

The application of the von Thünen and the Faustman approaches separately or in combination to economic models of shifting cultivation has been very limited so far (no attempts are known to the author). Models which combine these two approaches when it comes to forest used for timber production exist, for example in Ledyard and Moses (1976). By combining these two approaches, it is possible to make a more realistic description of shifting cultivation systems, and at the same time draw on the large literature that exists, particularly in the Faustman tradition. Thus, a contribution of this paper is partly to integrate these two approaches in general, and to apply them to a number of different settings for shifting cultivation in particular.

1.4 Labour market assumptions

Economic models for the study of agricultural decision-making can be categorized along a number of axes, in particular the behavioural and market assumptions (of which the labour, product, and credit markets are the most important). We focus on the labour market assumptions, for several reasons: They are closely connected with the behavioural assumptions that can be made (see below); they are crucial for the formulation and structure of the model; and differences in how labour markets function are a very distinct empirical feature. Four important and somewhat stylized categories of economic models for the study of agricultural decision making, which especially relate to the labour market assumptions and how the wage rate is determined in the model, are:[4]

1. *Small, open economy models*: Markets exist, and all prices (including the wage rate) are taken as parametrically given. An intuitive interpretation is that the shifting cultivation sector is small compared to the rest of the economy. In addition to the simplification made by exogenous prices, a further simplification is due to the recursive property of such models: If labour can be sold or hired at a constant wage, the production decisions by a *utility* maximizing household can be studied as income or *profit* maximizing production behaviour (Singh et al., 1986).[5]
2. *General equilibrium models:* Models where markets exist, and prices are determined endogenously, would in most cases provide a more realistic description than subsistence or open economy models, but a price is paid in terms of complexity. Coxhead and Jayasuriya (1994) provide one of the very few applications of this approach to environmental degradation in developing countries.
3. *Closed economy models:* No off-farm employment is available, and family labour is the only input in addition to land.[6] Product markets may or may not exist. In the latter case farmers produce only for their own consumption. We distinguish between two important versions of the closed economy model, based on differences in the behavioural assumptions:
    a. A common version is the *subsistence or "full belly" model*[7], e.g., Dvorak (1992). Farmers' objective is to meet a basic subsistence requirement, and they do so by minimizing their labour efforts (maximizing leisure).
    b. The *Chayanov (1966) model* is a more general formulation. The household acts as if maximizing a utility function, with consumption and leisure as the arguments. They reach a subjective equilibrium with a shadow wage rate reflecting the rate of substitution between consumption and leisure. In this way the Chayanov model resembles the general equilibrium model; a shadow wage is determined endogenously within the household (not in the market, as in 2.). Holden (1993) compares the "full belly" and Chayanov formulation in a study of shifting cultivation in Zambia.

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[4] This list of different categories of models is not exhaustive.

[5] The wage rate in the small, open economy model could well be the *expected* wage rate in the urban sector in a Harris-Todaro (1970) model.

[6] A situation when a fixed amount of off-farm employment becomes available is equivalent to a population change in the closed economy model.

[7] The term "full belly" is due to Fisk (1962).

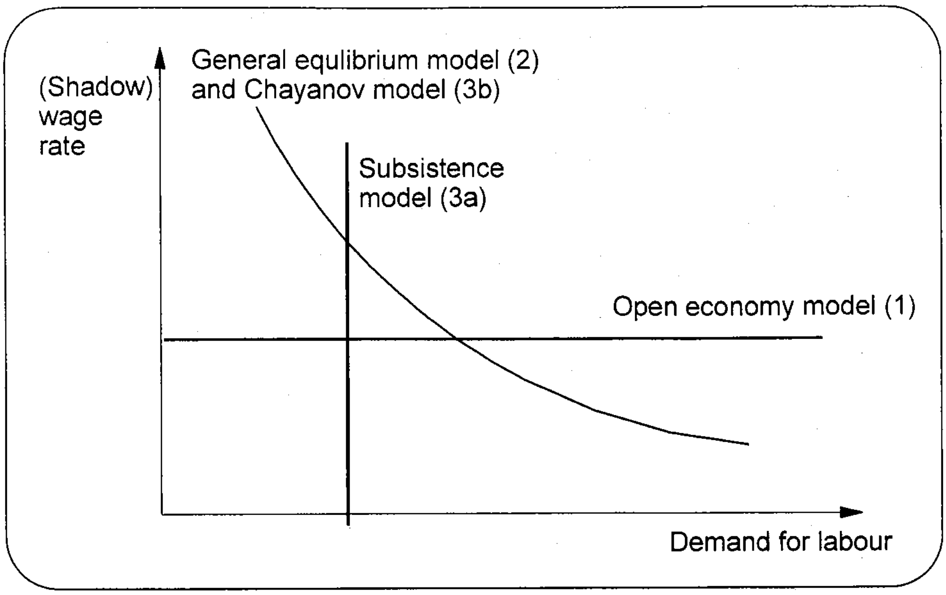

Figure 1. Labour market assumptions in four stylized economic models.

The difference between the various model categories with respect to the labour demand assumptions is illustrated in Figure 1.[8] This paper deals with the simplest case where the wage is fixed (small, open economy model). A companion paper deals with the closed economy model. Though the models in *2* and *3b* are more realistic formulations, and *1* and *3a* can be viewed as special cases of these two, they are analytically more complex. Moreover, as visualized by Figure 1, the assumptions in *1* and *3a* represent the two extreme cases, and therefore give the range of possible adaptations and responses to changes in exogenous factors.

When it comes to the output market, we assume output prices to be exogenously given (small, open economy). The credit market only enters the discussion implicitly. The recursive property of open economy models allows us to neglect the consumption side when analyzing production decisions. Thus the use of credit to, for example, smoothen consumption is not included. Neither are there any capital investments in the production, as labour and land are the only inputs. Moreover, we mainly confine the discussion to long-run steady states, where consumption equals income. To the extent a credit market is needed, the implicit assumption is that it works perfect; farmers can borrow and save as much as they want at a fixed interest rate (e.g., in the Faustman model). Though this may be unrealistic, we consider it to be of minor importance for the main arguments of the paper.

Which model gives the most realistic description of farmers' adaptation and responses to exogenous changes? It is commonly argued that the subsistence model may be the most appropriate for traditional societies, whereas the open economy models give a better description of a modernized society, e.g., Stryker, 1976.

[8] Note that the demand for labour in the figure is *total* demand (farm and off-farm).

Whereas Stryker and others focus on the behavioural assumptions, we argue that the labour market assumptions are equally, or even more important.

The appropriateness of the different models also depends on the time perspective. As an example, the open economy model, which assumes a migration equilibrium based on real wages in different sectors, is a more realistic description of the long term adaptation than of the short run. Another dimension necessary to consider is the unit or area studied; assuming prices to be fixed and determined exogenously may be more realistic for micro studies of individual decision-makers than for macro studies of a region or country. Definite tests of the subsistence versus the open economy hypothesis are difficult to formulate, and are rarely undertaken in empirical work (López, 1992). Moreover, one should keep in mind that these models are stylized descriptions, and empirical analysis may need to draw on elements from several approaches.

1.5 Property rights regimes

It is widely recognized that the property rights regime is a crucial factor in determining resource allocation in tropical agriculture in general, and in frontier systems, like shifting cultivation, in particular (see, for example, Bromley, 1991). The property regime is crucial in determining which costs and benefits that are to be included in the decision makers' optimizing problem. We can identify at least five different regimes or solutions to the model:

1. *Global social planner:* All externalities are included in the optimization problem.
2. *Communal management (local social planner):* Local, but not national or global, externalities are included.
3. *Private property:* No externalities are included, but discounted future private benefits and costs are included in the optimization problem.
4. *Open access:* Neither externalities nor any future benefits and costs are included.
5. *Homesteading:* This could be regarded as a special kind of open access, where forest clearing gives private property rights to the cleared land. Under this regime land is transferred from an open access resource (regime 4) to a private property resource (regime 3).

The global social planner's solution is employed to define the socially optimal solution, and acts as a yardstick to measure the outcome under other regimes. Each of the four other property regimes have empirical relevance, and will be discussed. State property is sometimes referred to as a separate property regime (e.g., Bromley, 1991). We have not included it as we could regard it as a special case of private property, where the owner is not a person, a household, or a firm but the state.[9] Parts of economic theory have traditionally not distinguished between state property and the social planner's solution, but little knowledge about tropical resource management is needed to realize the lack of realism in this assumption. One may hope, however, that state management would include at least *some* of the elements included in the social planner's problem. Generally, however, the state (or powerful groups within the state) may have strong financial interests in *certain* productive (as opposed to protective) uses of the forest, for example in logging or plantations.

Much of the debate on tropical deforestation and shifting cultivation is focused on environmental externalities like the carbon storage of tropical forest, and the preservation of biodiversity. We shall *not* pay too much attention to these issues (except some under the social planner's solution), not because they are unimportant, but because the model here will not add much to the standard approaches in environmental economics. Under all the four property regimes above (2.-5.), there will be no incentives to include (global) externalities, and the rate of deforestation will be too high. We do not make any attempt to answer the question of how much deforestation is optimal, that is how different uses of the forest should be balanced. Instead, a major aim of the paper is to explore which factors determine the expansion of shifting cultivation (extent of deforestation), and thereby identify policy handles which can be used to influence shifting cultivators decision-making.

The outline of the rest of the paper is as follows. In section 2 the main components of the model are developed. Section 3 discusses the simplest version of the social planner's solution, that is the single rotation (Fisher) model without discounting. Section 4 deals with the more complete multi-rotation (Faustman) problem, where discounting and the value of land after clearing are included. Section 5 very briefly compares the Faustman solution with the communal and private management outcomes. The open access case is discussed in section 6. Section 7 deals with a special case of open access, namely when forest clearing gives property rights to the farmers (homesteading). Section 8 compares the solution of the different models, and the effects of changes in exogenous variable. Section 9 provides some concluding remarks.

[9] Communal property could indeed also be considered a special case of private property, where the owner is a group of individuals, e.g., a community. The main distinction is between situations *with* property rights (where the agent with the rights is either the almighty, fully informed, and welfare maximizing social planner; the community; the state; or an individual/household), and situations where *no one* has property rights (open access). Real life situations will be a continuum along this axis, depending on how *secure* the rights (claims) are. Another complication of this categorization is the fact that the agent may not be well defined, for example, individual households may use land in a particular way after consultations with the leaders of the community. Property rights are a bundle of rights, which are always constrained to various degrees, for example, households may not be allowed to sell the land (to outsiders). Finally, a resource (land) may have different regimes governing different uses, for example, agricultural use resembles a private property regime, whereas collection of forest products from the same land is governed by communal management.

The present, theoretically oriented paper is complementary to Angelsen (1994). The latter gives a non-technical analysis of the main factors behind shifting cultivation expansion, and a presentation and analysis of recent changes in the shifting cultivation system of the Seberida district, Sumatra.

2 Basic model

2.1 Fallow period and intensity of production

A crucial variable in a shifting cultivation system is the length of the fallow period, or to be more precise: The relationship between the fallow period and the cropping (tillage) period. Let H be cropping land, A total agricultural land (cropping and fallow land), C the length of the cropping cycle, and F the length of the fallow period. Then we have the following relationship;

$$(1) \qquad A = \frac{H}{\frac{C}{C+F}} = Hm \quad \Leftrightarrow \quad \frac{H}{A} = \frac{1}{m}$$

Here $\frac{C*100}{C+F} = R$ is Ruthenberg's (1980) R-value, i.e., the percentage of land that is under cultivation. $m = \frac{C+F}{C}$ is Boserup's (1965) land use intensity factor, which will be the key variable in our model. The inverse of m gives the share of land under cultivation, and can be used as a measure of intensity of production; lower m implies an intensification. Indeed, agricultural systems are commonly classified on the basis of these factors, as done by Boserup (1981: 19): Forest fallow (R = 0 - 10); bush fallow (R = 10 - 40); short fallow (R = 40 - 80); annual cropping (R = 80 - 100); and multicropping (R = 200 - 300).[10] Ruthenberg (1980: 16), on the other hand, distinguishes between shifting systems ($R < 33$); fallow systems ($33 < R < 66$); and permanent cultivation systems ($R > 66$).

If we set the cropping period (C) to unity, $m = (1 + F)$. In the following we shall for simplicity (to make the language easier) refer to m as the fallow period, but keeping in mind that m is actually the length between the beginning (or the end) of two cropping cycles.

2.2 Production function

The yield or output per ha of cleared land or land currently in production (x) is dependent on the length of the fallow period (m), the labour inputs for weeding, pest control, etc. (l), and the technology level (a).[11] Labour for clearing is determined by the fallow period (see below), and is not a choice variable and has no yield effect in the model.

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[10] A more general definition of R is to multiply in the above definition by the number of harvests per year, thus the R-value exceeds 100 if there is more than one harvest per year.

[11] The formulation partly follows the function used by Dvorak (1992): $x = f(C,F,l)$, where F is the fallow period, and C the cropping period (number of years crops are grown between fallow).

(2) $x = af(m,l); \quad f_m \geq 0, f_{mm} \leq 0, f_l \geq 0, f_{ll} \leq 0, f_{ml} = f_{lm} \leq 0; a > 0$

Yield is an increasing function of the length of fallow, as longer fallow increases the biomass and thereby the fertilization of the soil through burning. Also, increasing m implies less weed and pest problems. The marginal increase in x is declining as m increases and eventually reaches a maximum ($f(\ )$ is concave). Similarly, the yield effect of increasing labour input is positive, but decreasing. The crossderivatives are assumed to be negative, i.e., the marginal productivity of labour decreases as the fallow increases, as, for example, weeds become less of a problem. This is in line with Dvorak (1992), whereas López and Niklitschek (1991) assume a *positive* crossderivative. An argument for a positive sign is the fact that increased fallow period means more fertile land, and this *could* increase the marginal return on labour. A third possibility is that the sign depends on the level of m, for example in the way that the crossderivative is *positive* for small values of m, whereas it is *negative* for large values of m. The empirical evidence to determine the sign is weak. In any event, one should try to avoid letting the sign of the crossderivative drive any major conclusions in the model. As the later analysis will show, none of the main conclusions on how m is affected by changes in exogenous variables depend on this assumption.

Technical change is represented in this model by the parameter a in a manner implying Hicks neutral technical progress. The main argument of Boserup (1965) and others is that most of the technical change in shifting cultivation system is *endogenous*, depending on particularly the fallow period, which in turn is determined by factors like the population pressure. The models presented in this paper, like most models for agricultural decision-making, do *not* include endogenous technical change. Technical progress included in a in our model could be for example better yielding crop varieties.[12]

Finally, this formulation of the production function implies that the elasticity for total production (X) with respect to land is one (cf. the assumption of homogenous land). $X = Haf(m,l)$, where H is the cropping area (land currently in production).

2.3 Labour costs

We include three types of cost in the model.[13] The first type of labour input is weeding, pest control, etc. described above. Second, labour for clearing and preparation of the field (g), which depends on the fallow period, $g = g(m)$, in that longer fallow requires more work to clear the field (larger trees to cut and burn).

[12] Even though the high yielding varieties (HYV) associated with the Green Revolution in intensive, irrigated agriculture is not very relevant to shifting cultivators, some intermediates between traditional crop varieties and HYV may be.

[13] Ruthenberg (1980: 50-51) separates the labour operations in shifting cultivation as follows: (1) Clearance of wild vegetation; (2) land preparation and planting; (3) weeding; and (4) harvest, transport of harvest, and processing. A slightly different categorization is used in this paper, which is more appropriate to the models developed.

The function *g()* reaches its maximum when the forest reaches its climax vegetation;

$$g = g(m), g_m \geq 0, g_{mm} \leq 0$$

Third, there are costs related to the location of the field, as measured by the distance from the village (*b*). These may be thought of as time (*c*) spent on walking between the fields and the village. A number of alternative formulations of the distance cost function is possible. We have chosen a specification which is both simple and have some intuitive appeal. It assumes *c* to be proportional to both distance and time working on the field per unit land *(l + g)*;

$$c = qb[l + g(m)]$$

q is the time spent on walking per km for one day of work on the field. Our formulation implies multiplicative distance costs, both in distance and in on-the-field labour inputs *(l + g)*. Thus, increased distance has exactly the same effect as a real wage increase in the model, which turns out to be a neat simplification. In reality there are both additive and multiplicative elements related to distance. If we have made distance costs only additive, an implication later would be that fallow length and labour inputs are independent of distance. This is clearly an unrealistic description which does not correspond to empirical observations. We have chosen to include only the multiplicative elements as these are the most important. Additive costs would only have implications for the determination of the agricultural frontier, whereas multiplicative costs are important for all three endogenous variables (labour, fallow, and agricultural frontier). Thus, adding an additive component does not give any new insight or change the main results.[14]

This formulation of distance costs also implies that there is no optimization of transport costs, for example, in the way that farmers would work more per trip on the distant fields. This is an argument for the costs being to be concave in distance. On the other hand, one may argue that time spent on walking per km should be convex in distance, e.g., one may need to take a rest on longer trips. All in all, the linearity assumption may mot be a perfect representation, but its simplicity and the lack of convincing arguments for a particular alternative make it acceptable.

Summarizing the three types of labour costs, we get;

(3) $$l + g(m) + qb[l + g(m)] = (1 + qb)[l + g(m)]$$

[14] Additive costs would behave like a kind of sunk costs in the model: They would be important to the decision of whether or not to open a swidden at a given distance, but afterwards they would *not* influence the decisions regarding fallow period and labour input.

2.4 Land rent

In a static model, the land rent (r) or profit from one single clearing of a plot at a given distance from the village, as measured in units of the agricultural product (numéraire), is given by;[15]

(4) $$r[m,l;a,w(1+qb)] = af(m,l) - w(1+qb)[l+g(m)]$$

w is the real wage rate, defined as nominal wage divided by the agricultural output price (i.e., the price of the agricultural output acts as a price deflator). Note that with our formulation of distance cost, these are equivalent to the more common formulation where the net output price is declining with distance due to costs of transportation of output.

The maximum (undiscounted) profit from a single clearing is found by setting $r_m = r_l = 0$ (FOC);

(5) $$\frac{f_m(m^*,l^*)}{g_m(m^*)} = f_l(m^*,l^*) = \frac{w}{a}(1+qb) = z$$

(5') $$m^* = m^*(z); l^* = l^*(z)$$

The second order condition, which ensures that (5) is a maximum point, is given by the assumption that $r(.)$ is concave in m and l: $r_{mm} < 0$, $r_{ll} < 0$, $r_{mm} r_{ll} - r_{ml} r_{lm} > 0$.

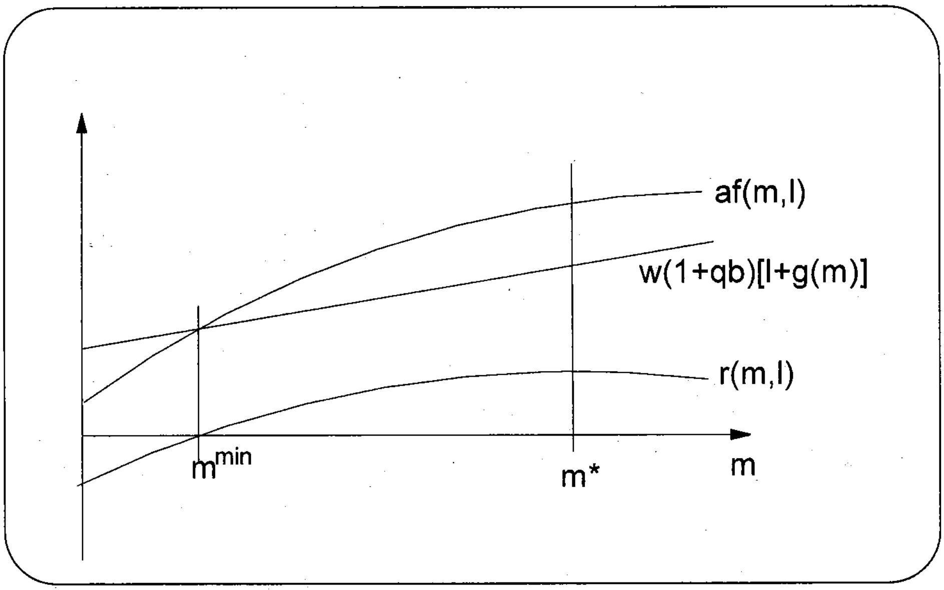

Figure 2. The optimal fallow period (m).[16]

[15] This model gives the maximum land rent from one clearing. It does not, *inter alia,* take into account discounting or the value of land after clearing, which is considered later.

[16] l would in general vary with m; higher m implies lower l because $f_{ml} < 0$. In drawing the figure

The optimal choice of m is illustrated in Figure 2. The assumption of $r_{mm} < 0$ implies that as m increases, the decline in marginal productivity (f_m) is larger than the decline in marginal costs (g_m). This would be true if there, for example, is a strictly concave relationship between yield and biomass whereas clearing costs are proportional to the biomass. The shape of $r(\ )$ is discussed in more details in section 7.5.

We note that all the exogenous factors can be summarized into one variable, $z = \frac{w}{a}(1+qb)$, which may be interpreted as the *effective real wage*, taking into account both the technological level and the distance costs. The effects of changes in z are explored later. The highest possible land rent and the effect of exogenous changes are then given by;

(6) $$r = af(m^*, l^*) - w(1+qb)[l^* + g(m^*)]; \quad r = \tilde{r}(a, w(1+bq))$$

(6') $$\frac{dr}{d[w(1+qb)]}\Big|_{da=0} = -[l^* + g(m^*)] < 0; \quad \frac{dr}{da}\Big|_{d[w(1+qb)]=0} = f(m^*, l^*) > 0$$

The results in (6') follow by applying Hotelling's lemma.

2.5 *Minimum fallow*

We define m^{min} as the minimum fallow period which gives a non-negative profit, as illustrated in Figure 2. We assume that this occurs for $m > 0$, which corresponds to the definition of a shifting cultivation system (often defined as $m > 2\text{-}3$, see Ruthenberg, 1980).

(7) $$f(m^{\min}, l^{**}) - z[l^{**} + g(m^{\min})] = 0; \quad m^{\min} = m^{\min}(z)$$

(7') $$\frac{dm^{\min}}{dz} = \frac{l^{**} + g(m^{\min})}{f_m(m^{\min}) - zg_m(m^{\min})} > 0; \quad b \in [0, b^{\max})$$

It follows from the definition of m^{min} that labour inputs must be chosen optimally according to $f_l = z$, and we have labelled the optimal labour input for the minimum fallow period l^{**} to distinguish it from the optimal labour input given in the problem in (5). One should note that the denominator in (7') is positive. Even though this resembles the first order condition in (5), here the expression is evaluated at $m = m^{min}$.

(7') shows that the minimum fallow is an increasing function of z, that is, increasing in distance (b), real wage (w), and travel efficiency (q), and decreasing in the technology level (a).

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we have neglected this feature, which is of less importance to illustrate the basic relationship.

2.6 Agricultural frontier

Finally, we define the agricultural frontier (margin of cultivation) or maximum distance (b^{max}) at which the land rent would still be non-negative, cf. Figure 3 below. Obviously, this will occur when the fallow period and labour inputs are optimally chosen according to (5).

(8) $$f(m^*,l^*) - \tfrac{w}{a}(1+qb^{\max})[l^* + g(m^*)] = 0$$

$$\Leftrightarrow b^{\max} = \frac{f(m^*,l^*)}{\frac{w}{a}q[l+g(m^*)]} - \frac{1}{q}; \quad b^{\max} = \tilde{b}(\tfrac{w}{a},q)$$

(8') $$\frac{db^{\max}}{d\frac{w}{a}}\Big|_{dq=0} = -\frac{1+qb^{\max}}{q\frac{w}{a}} < 0; \quad \frac{db^{\max}}{dq}\Big|_{d\frac{w}{a}=0} = -\frac{b^{\max}}{q} < 0$$

The maximum distance at which shifting cultivation will take place is negatively related to the real wage (w), positively to the technical level (a), and negatively to the travel efficiency factor (q). We note that the minimum fallow equals the optimal fallow for plots located at the agricultural margin ($m^* = m^{min}$ *at* b^{max}.)[17]

Figure 3 illustrates the determination of the agricultural frontier. The variables m and l are in general functions of b, and, for example, the *af(m,l)* - curve will in general not be horizontal. We have neglected this when drawing the figure as the sign of the relationship between m and b is different in the single and multi-rotation problem, as seen below.

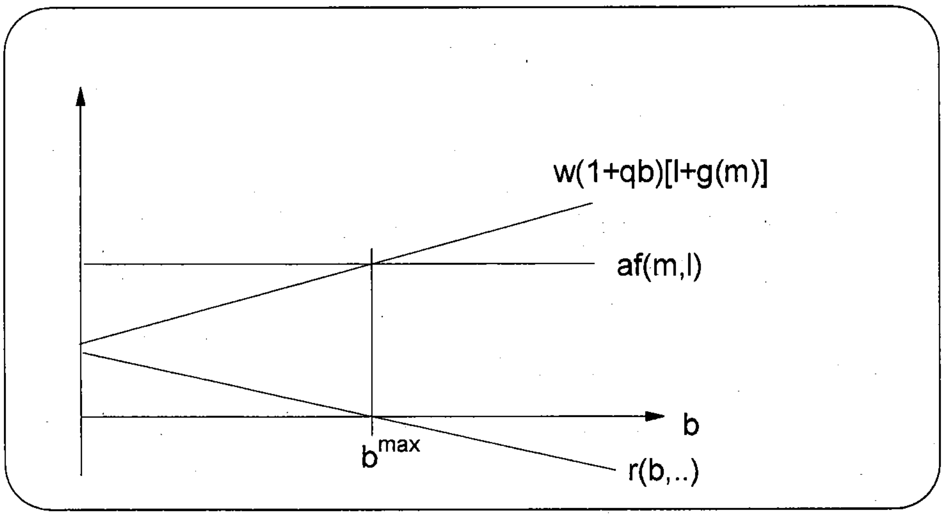

Figure 3. The determination of the agricultural frontier (b^{max}).

[17] One should note that there is no effect on b^{max} from the effect a change in z has on m and l (the envelope theorem).

3 Social planner's solution I: The single rotation problem (Fisher)

3.1 The model

We first consider the simplest version of the social planner's solution, where the objective is to maximize the net benefits from one rotation. Historically, this formulation goes back to Irving Fisher (1907) who uses a one-rotation forestry management problem in his discussion of capital theory (see for example Hartwick, 1993, for a discussion).[18] Though this is based on unrealistic assumptions, it is illustrative as a first case, and serves to contrast the basic characteristics of the multi-rotation problem. It is customary to present both cases in the literature, see for example Hartman (1976), Heaps and Neher (1979), Hartwick (1993), Reed and Clark (1990). We present a special case of this problem, that is when there is no discounting. The case with discounting is discussed in Appendix 3.

The problem to the social planner is to maximize total land rent of all land from one rotation. We assume to start from bare land, that is when all forest is of age *0*. As the time horizon is one rotation, we neglect the value of land after the clearing and when cropping is over. Because of our assumption about a village surrounded by homogenous land, only differing in distance from the village, the area of cultivation will be a circle around the village. This assumption simplifies the analysis, and produces some interesting results. The analysis is also valid for cases where the land available is a *fraction* of a circle.

Total land rent (TR) from all plots is to be maximized with respect to labour inputs, fallow period (when to cut?), and by determining the agricultural frontier (b^{max}).

(9) $$Max_{m,l,b^{\max}}\ TR = max_{m,l,b^{\max}} \int_0^{b^{\max}} \{af(m,l) - w(1+qb)[l+g(m)]\}2\pi b\, db$$

The expression in {} gives the rent from one clearing. This is integrated over total area cleared, where $2\pi b$ is the circumference of a circle with radius b.

The FOC are;

(10) $$\frac{\partial TR}{\partial m} = \int_0^{b^{\max}} \{af_m - w(1+qb)g_m\}2\pi b\, db = 0 \Leftrightarrow f_m - zg_m = 0;\ for\ b \in [0, b^{\max}]$$

(11) $$\frac{\partial TR}{\partial l} = \int_0^{b^{\max}} \{af_l - w(1+qb)\}2\pi b\, db = 0 \Leftrightarrow f_l - z = 0;\ for\ b \in [0, b^{\max}]$$

(12) $$\frac{\partial TR}{\partial b^{\max}} = \{af(m,l) - w(1+qb^{\max})l + g(m)]\}2\pi b^{\max} = 0 \Leftrightarrow r = 0 \quad at \quad b = b^{\max}$$

We see that the solution given by (10) - (12) is the same as (5) and (8). The assumption of $r()$ being concave in m and l ensures that we can use "⇔" in (10) -

[18] The solution to this problem is also called the Wicksell-Fisher method (Manz, 1986: 284).

(12). The interpretation of these conditions are straightforward: Labour is chosen in such a way that marginal productivity equals effective real wage (z), that is, the real wage adjusted for the time spent on travelling between the village and the field. The fallow period is similarly chosen in such a way that the production increase of longer fallow equals the increased clearing costs of extending the fallow period. Finally, cultivation is expanded from the village in such a way that the rent at the agricultural frontier is zero. Note that because the fallow period and the labour input are functions of *z*, the optimal values of *m* and *l* will vary with distance, as shown under the comparative statics below.

The system (10) - (12) is partly recursive; (10) and (11) give the optimal values of *m* and *l*, which, inserted in (12), give the optimal choice of b^{max}. Thus, the problem can be disaggregated into maximizing the benefit from each plot, as there is no overall production target to be met or other connections between the different plots (e.g., external effects). The optimal choices of *m* and *l* are illustrated in Figure 4 below.

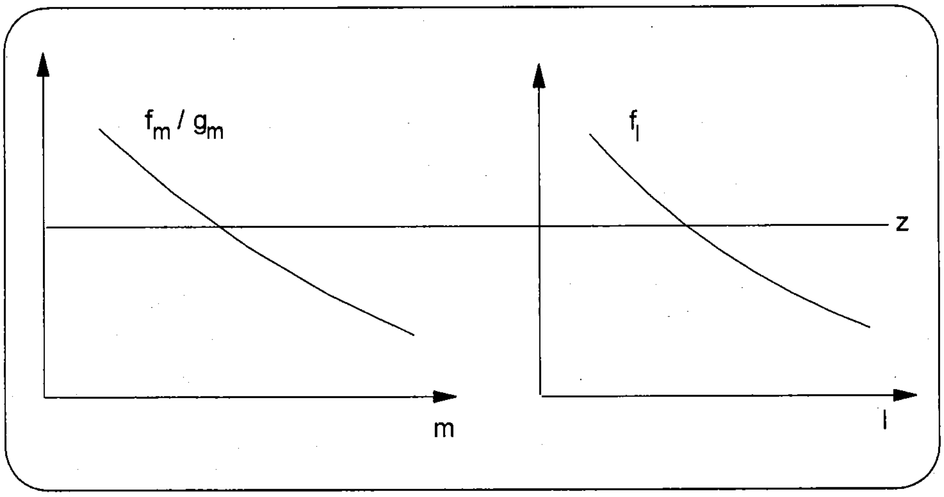

Figure 4. The determination of fallow period (m) and labour input (l).

3.2 Comparative statics

Using the property of the solution that each individual plot can be looked at in isolation (maximizing total rent is the same as maximizing rent from each plot in isolation), we want to explore the effects on *m* and *l* of changes in *z*. Differentiation of the FOC in (5), or (10) and (11), yields;[19]

$$(13) \quad \frac{f_{mm} g_m - f_m g_{mm}}{g_m^2} \frac{dm}{dz} + \frac{f_{ml} g_m}{g_m^2} \frac{dl}{dz} = 1 \quad or \quad a_{11} \frac{dm}{dz} + a_{12} \frac{dl}{dz} = 1$$

[19] The a_{ij} used here as a shorthand notation for the expressions in (13) and (14) should not be confused with the parameter *a* used for the technological level.

(14) $f_{lm}\frac{dm}{dz}+f_{ll}\frac{dl}{dz}=1 \quad or \quad a_{21}\frac{dm}{dz}+a_{22}\frac{dl}{dz}=1$

The second order conditions (SOC) for (10) and (11) to be a maximum point require the determinant of this system (13) and (14) to be positive;

(15) $D=a_{11}a_{22}-a_{12}a_{21}>0$

In addition, the SOC consists of $r_{mm} < 0$ (or $r_{ll} < 0$).[20] It is easy to verify that the condition in (15) is assured by the assumption that *r( )* is concave in *m* and *l*. The effect of a change in *z* can be found by application of Cramer's rule;

(16) $\frac{dm}{dz}=\frac{a_{22}-a_{12}}{D}<0$

(17) $\frac{dl}{dz}=\frac{a_{11}-a_{21}}{D}<0$

a_{12} , a_{21} and a_{22} have been assumed to be negative under our discussion of the production function. a_{11} is also assumed to be negative, corresponding to the SOC for maximum. Finally, we assume that a_{12} and a_{21} do not dominate in a_{11} and a_{22} in (16) and (17), respectively. Thus, both *m* and *l* will decrease as *z* increases. Note that none of these results depend on the assumption of $f_{lm} = f_{ml} < 0$, in fact, if we assume the crossderivatives to be positive, we can safely conclude that *both* (16) and (17) are positive.

As shown in Appendix 1, the SOC ($D > 0$) ensures that at least one of the expressions in (16) and (17) is negative, but we have just assumed that both are. The interpretation of this assumption is as follows. The first order effect of an increase in *z* is given by an upward the shift in the *z-curve* in Figure 4. This will reduce both *m* and *l*. However, changes in these two variables will shift the other curve in both diagrams upward, which will increase both *m* and *l*. We presuppose that this second order effect does not outweigh the first order effects for neither *m* nor *l*.

We have now established that the fallow period and labour input will *decrease* with distance from the village. Further, an exogenous increase in the real wage will reduce the fallow period and labour input, whereas technological progress and improved transportation efficiency or accessibility will have the opposite effect.

The model produces a somewhat surprising result, namely that the fallow period is becoming shorter (higher intensity of production) the further away from the village the land is. This result may be both counterintuitive and against the empirical evidence, and should be understood in terms of the assumptions made and the particular model specification. The fallow length (*m*) in the *single* rotation model is handled in the same way as a standard factor of production in the neo-classical

[20] It it easy to verify that given $D > 0$, and $r_{mm} < 0$, it follows that $r_{ll} < 0$.

analysis of the firm. The costs are increased labour for clearing; the benefits are increased fertility and production. The marginal net benefits are declining *(*r_{mm} *< 0)*, and an increase in the costs of clearing (real wage) will make a rational planner shorten the fallow period (use less of the input). The single rotation model does *not* capture important aspects related to the opportunity costs of land, as will be seen in the multi-rotation model in the next section. This, of course, questions the applicability of the model. Indeed, Paul Samuelson labels the single rotation model "Fisher's false solution" (Samuelson, 1976: 470). In addition, we have assumed no discounting. Appendix 3 shows that if the discount rate is sufficiently high, the sign in (16) may be reversed.

From (16) and (17) we also get that a real wage increase leads to an intensification of the system in terms of shorter fallow period, whereas labour inputs decrease, which also may be doubtful (see for example Boserup, 1965). This result is related to the above one as increased distance has exactly the same effect as a real wage increase in the model, because distance costs are multiplicative to on-the-field labour inputs $(l + g)$ and distance.

What happens to the agricultural frontier when z increases? We can apply the result in (8') directly: A decrease in w or q, or a rise in a will increase b^{max}. The margin of cultivation is determined by the relative profitability of shifting cultivation, and any change in exogenous factors which increase the profitability will expand the area under shifting cultivation.

3.3 Land rent and transport costs

A further look at the agricultural rent yields some interesting results, which relates to a debate in urban economics on the relationship between land rent and transport costs (see for example Arnott and Stiglitz, 1979). Integration by parts of (9) gives;

$$(18) \qquad TR = \{af(m,l) - w(1 + qb^{\max})[l + g(m)]\}\pi(b^{\max})^2 + \int_0^{b^{\max}} wq[l + g(m)]\pi b^2 \, db$$

In general, (18) splits the overall rent into two different types. The first term is the rent at the agricultural margin, multiplied by the total area under cultivation. This is the *scarcity rent.* In our case, where we have not imposed any physical restrictions on agricultural expansion, land rent at the margin is zero. Thus, the scarcity rent is zero in our model. In fact, Arnott and Stiglitz (1979: 473) use a zero land rent at the border as a definition of land abundance. The second term in (18) is the *differential rent.* This is rent due to the fact that land has different locations, and there are costs related to a distant location. All land except at the frontier have a positive differential rent, which is inversely related to how close land is to the village.

Total transport costs (*TTC*) are given by;

(19) $TTC = \int_0^{b^{max}} wqb[l + g(m)]\, 2\pi b\, db$

Comparing (18) and (19), we see that the differential rent is half of the total transport costs.[21] The result leads to the somewhat counter-intuitive conclusion that reduced *TTC* actually *reduces* land rent. Generally, economic intuition suggests that reducing the costs should increase the rent. Two points are important to understand the conclusion we get in this case. First, one must keep in mind that the only land rent here is the differential rent, which is due to costly transport. When *TTC* decreases, the importance of location is reduced, and the land rent will decline. Second, it is crucial to distinguish between *TTC* and transport costs per km per working day on the field, that is q. The discussion of this result in, for example, Hartwick and Olewiler (1986: 46) may leave the reader confused, and does not tell the whole story. To see what happens when q changes, we take the partial derivative of (18) with respect to q, using Leibniz' formula.

(20) $\frac{\delta TR}{\delta q} = \int_0^{b^{max}} w[l + g(m)]\pi b^2 db - wq[l + g(m)]\pi (b^{max})^2 \frac{b^{max}}{q}$

$$= w[l + g(m)]\pi \frac{(b^{max})^3}{3} - w[l + g(m)]\pi (b^{max})^3 = -w[l + g(m)]\pi \tfrac{2}{3} (b^{max})^3 < 0$$

The effect of a decrease in q is split into two terms in (20). First, the differential rent on land within the old border will decline, which is the story just told. The second term captures the fact that the agricultural frontier will expand, where we have used our result from (8') for the effect on b^{max} of a change in q. This has a positive effect on the land rent. Summarizing the two effects, we see that the net effect of a decline in q on total land rent is positive, contrary to what the first result indicated. As the above result about the relationship between total land rent and total transport costs still holds, of course, we have found that a *reduction* in the per km transport costs (q) will actually *increase* the total transport costs. The decline in q leads to an expansion of the frontier that more than outweighs the reduced transport costs on the area within the old border.

3.4 Summary & conclusions of the single rotation model

- The optimal fallow period is decreasing in the real wage rate and distance from the village, but increasing with travel efficiency and technological level.
- Labour inputs, other than for clearing, will be declining if we have an increase in the real wage or in the distance, and increasing with technological progress or improvements in travel efficiency.
- The agricultural frontier is positively related to the technological level and the travel efficiency, but negatively to the real wage.

[21] This result has an intuitive geometric interpretation (see Mohring, 1961, or Arnott and Stiglitz, 1979, footnote 5).

- Better off-farm employment opportunities which result in higher real wage will lead to shorter fallow periods and less labour inputs, and reduce the agricultural frontier and total area under cultivation.
- Technological progress or higher agricultural price (lower w) will have the opposite effect, and lead to longer fallow periods, more labour inputs per ha, and an expansion of the agricultural frontier.
- Improvements in travel efficiency, e.g., by new roads, will have the same effect as technological progress.
- The effect on land rent of improvements in travel efficiency (lower travel costs per km) is positive. It also increases total transport costs, which are twice the size of the rent.

4 Social planner's solution II: The multi-rotation problem (Faustman)

4.1 The model

The single rotation model just presented overlooks two important factors which should enter the decision making of a social planner. First, the time horizon is one rotation, and land is assumed to have no value after the (first) rotation is completed. It does not take into account the opportunity cost of land: When forest is cleared and the cropping period is over, a new cycle can start. Thus, there is a cost of delaying clearing and cultivation. Second, it does not include discounting, or to be more precise: The above model implicitly assumes a zero discount rate. If there is a positive discount rate, there is a cost of delaying clearing and cultivation, because the benefits (harvest) will be postponed. Both these effects push the solution towards shorter rotation period, as will be seen more clearly below.

The solution to the multi-rotation problem when it comes to timber production goes back to Faustman (1849), a remarkably early statement of what remains the basic formula for most analysis in forestry economics and capital theory one and a half century later.[22] The presentation here draws on Clark (1990), Hartman (1976) and others.

We make three basic assumptions:

1. The problem is to maximize the net present value (*NPV*) of land rent from the total agricultural area, which is endogenously determined.
2. All parameters (prices, discount rate, technology, and functional forms) are known and remain constant over time. This is clearly the most critical assumption, and will be discussed later.

[22] The solution to the multi-rotation problem has different names in the literature; the Faustman-Ohlin theorem (Löfgren, 1983) and the Faustman-Hirshleifer-Samuelson optimization (Manz, 1986) being two of them.

3. The time horizon is infinite, which -- together with the second assumption -- simplifies the analytics significantly as we may use the formula for an infinite geometric series to transform the problem into one where we can use static optimization.

From these assumptions it follows immediately that all fallow periods will be of the same length (for a given distance from the village). We shall mainly deal with long term equilibria or steady state solutions, and do not discuss the path between different steady states.

The model will still be recursive, as under the single rotation problem; first optimal fallow and labour inputs are determined, then the agricultural frontier. The maximum *NPV* or discounted land rent (equal the land price in a competitive economy) for a plot at a given distance ($b \le b^{\max}$) can now be written as;[23]

$$(21) \quad Max\ NPV_{m,l} = \max\ \Sigma_{j=1}^{\infty} e^{-jm\delta}\{af(m,l) - w(1+qb)[l+g(m)]\}$$

$$= \max \tfrac{1}{e^{m\delta}-1}\{af(m,l) - w(1+qb)[l+g(m)]\}$$

The FOC are given by;

$$(22) \quad \tfrac{\partial NPV}{\partial m} = \tfrac{-e^{m\delta}\delta}{(e^{m\delta}-1)^2}\{af(m,l) - w(1+qb)[l+g(m)]\} + \tfrac{1}{e^{m\delta}-1}\{af_m - w(1+qb)g_m\} = 0$$

$$\Leftrightarrow (1-e^{-m\delta})[af_m - w(1+qb)g_m] - \delta[af() - w(1+qb)(l+g)] = 0$$

$$\Leftrightarrow \tfrac{af_m - w(1+qb)g_m}{af() - w(1+qb)(l+g)} = \tfrac{\delta}{1-e^{-m\delta}}$$

$$\Leftrightarrow af_m - w(1+qb)g_m = \delta[af() - w(1+qb)(l+g)] + \tfrac{\delta[af() - w(1+qb)(l+g)]}{e^{\delta m}-1}$$

$$or \quad r_m = \delta r + \tfrac{\delta r}{e^{\delta m}-1}$$

$$(23) \quad \tfrac{\partial NPV}{\partial l} = \tfrac{1}{e^{m\delta}-1}\{af_l - w(1+qb)\} = 0 \Leftrightarrow f_l - z = 0\ or\ r_l = 0$$

(23) is similar to the condition (11) in the single rotation problem. The new condition is the Faustman formula in (22). The last line in (22) is the one which gives the clearest economic interpretation of the Faustman result. The LHS gives the benefits in terms of increased net yield from one clearing by postponing forest clearing for one year. At the optimum, this should equal the costs of one year delay: The first term on the RHS is a *capital cost*, that is the cost that is incurred by delaying the profit by one year. This equals the rent from one clearing times the interest or discount rate. The second term is the opportunity cost of land, or *site*

[23] This is only valid for positive discount rates, the case of a zero discount rate is discussed as a special case below.

value. $\frac{1}{e^{m\delta}-1}r$ is the present value of future net benefits, which when multiplied by the discount rate, gives the cost of delaying clearing by one year.

How does the multi-rotation (MR) solution compare with the single rotation (SR)? The optimality condition under the single rotation problem in (10) implies setting the RHS of (22) to zero. Due to the concavity of *r( ),* the multi-rotation problem gives a shorter fallow period. In the single rotation problem there are no costs of delaying clearing, whereas the Faustman-formulation introduces two types of cost, as described above.

It is also clear that labour inputs (*l*) will be higher under the multi-rotation problem. This is readily apparent from (23), and given that $m^{SR} > m^{MR}$ and the assumption $f_{lm} < 0$. A decline in m will increase the marginal productivity of labour, *ceteris paribus,* and therefore increases labour efforts.[24]

The agricultural frontier is determined in the same way as in the single rotation model, i.e., the rent at the margin should be zero. Formally, we have (where m^{MR} and l^{MR} indicate optimal values under the multi-rotation problem);

$$(24) \quad \frac{\partial NPV}{\partial b^{\max}} = \frac{1}{e^{m^{MR}\delta}-1}\{af(m^{MR}, l^{MR}) - w(1+qb^{\max})\{l^{MR} + g(m^{MR})\} = 0$$

$$\Leftrightarrow r = 0 \quad at \quad b = b^{\max}$$

The comparison of the two cases with respect to the agricultural frontier is straightforward. Remembering that the frontier is defined where the land rent is zero, the RHS in the last equation in (22) equals zero, thus it equals (10). At the margin, the fallow period will be the same in both models (but shorter in the multi-rotation than in the single rotation model for land inside the margin). With m and l the same and $r = 0$ in both models, the frontier will be the same in the two models. Consequently, the effect of a change in z on b^{max} would be the same in the two cases.

4.2 Comparative statics

The model in (22) - (24) has three endogenous variables, *m, l* and b^{max}, and two exogenous variables, z and δ. The recursive property simplifies the comparative statics. Differentiation of the first two equations yields;[25]

$$(26) \quad [(1-e^{-\delta m})(r_{mm} - \delta r_m)]dm + [(1-e^{-\delta m})r_{ml} - \delta r_l]dl$$

---

[24] The comparison of the two models is somewhat more complicated that indicated here, because (22) and (23) are simultaneous equations, e.g., the increase in l has an effect on m. However, we have assumed here that the direct effects dominate the indirect, as done for comparative statics elsewhere.

[25] Again, the b_{ij} used below should not be confused with the b used as a symbol for distance.

$$= [r - me^{-\delta m} r_m] d\delta + [(1 - e^{-\delta m}) g_m - \delta(l + g)] dz$$

$$or \; a_{11} dm + a_{12} dl = b_{11} d\delta + b_{12} dz$$

(27) $f_{lm} dm + f_{ll} dl = dz; \; or \; a_{21} dm + a_{22} dl = dz \; (b_{21} = 0, b_{22} = 1)$

First we see that all $a_{ij} < 0; i,j = 1,2$, and we assume that the determinant is positive, $D = a_{11}a_{22} - a_{12}a_{21} > 0$, corresponding to the SOC for maximum. We also have $b_{11} > 0$, because $r \geq mr_m$ (concave) and $e^{-\delta m} < 1$. As seen from (22') below, it goes to zero as the discount rate approaches zero. Finally, we show in Appendix 2 that we always have $b_{12} < 0$.

We then obtain;

(28) $\frac{dm}{dz} = \frac{1}{D}[a_{22}b_{12} - a_{12}b_{22}] > 0$

(29) $\frac{dl}{dz} = \frac{1}{D}[a_{11}b_{22} - a_{21}b_{12}] < 0$

(30) $\frac{dm}{d\delta} = \frac{1}{D}[a_{22}b_{11}] < 0$

(31) $\frac{dl}{d\delta} = \frac{1}{D}[-a_{21}b_{11}] > 0$

An increase in *z* implies a longer fallow period, thus our specification of the model confirms some common results in forestry economics: A price increase leads to shorter rotation period, whereas an increase in the wage level has the opposite effect (Hyde and Newman, 1991:85). Increased rotation when the (net) price increases is known as the *"Ricardo effect"* in capital theory (Ledyard and Moses, 1976: 151). As a corollary, (28) also implies that the fallow period will increase with distance from the village, a result corresponding to empirical observations in many tropical regions (e.g., Hiraoka, 1986; Angelsen, 1993) and other theoretical models (e.g., Heaps, 1981, proposition 5).

Furthermore, an increase in *z* also implies less labour efforts for two reasons (29): The first one is the standard effect of a higher real wage. Second, the marginal productivity of labour declines due to longer fallow periods, even though this effect is based on rather weak empirical foundations, as discussed above. This also implies that labour per ha declines with distance from the village, a result which is in line with Ledyard and Moses (1976) and others.

A higher discount (interest) rate results in a shorter fallow period (30), implying increased labour efforts because $f_{lm} < 0$ (31). The shortening of the fallow period follows intuitively from the logic behind the Faustman formula: A higher discount rate means that both the capital cost and the opportunity cost of land are higher, thus forest will be cut earlier.

We have earlier shown that the agricultural frontier will be the same in both the single- and multi-rotation, thus the effects of a change in z on b^{max} would be the same in the two cases. Moreover, a change in the discount rate does not affect the agricultural frontier in this model, as seen from (24) and (8).

4.3 Comparing the single- and multi-rotation solutions

We note that the effect on the fallow period of an increase in the real wage or distance is the opposite of that in the in single rotation model. The difference between the two models is illustrated in Figure 5 below. For simplicity we ignore the effects of changes in l due to changes in z for the time being, and concentrate on how m is determined.

In the single rotation model the FOC is $r_m = 0$, i.e., where the downward sloping curve intersects with the x-axis. The multi-rotation (MR) model includes two kinds of costs of delaying forest clearing, represented by the upward sloping curve. The optimal fallow period in the MR-model is where the marginal benefit curve (r_m) intersects with the marginal costs curve. We immediately see that the optimal fallow period is shorter in the MR problem.

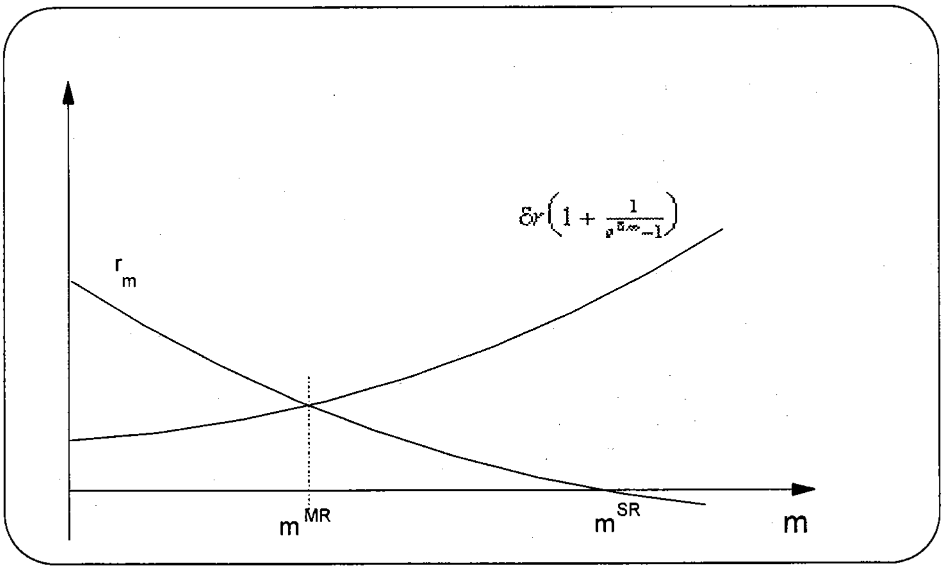

Figure 5. Comparison of the fallow period in the single rotation (SR) and the multi-rotation (MR) problem.

What happens when z increases? Both curves shift downward. In the single rotation problem, we see that the fallow period will be shorter. In the multi-rotation case, we have shown in Appendix 2 that $b_{l2} < 0$, which is to say that the shift in the upward sloping curve dominates. Thus, the effect of an increase in z is to increase the optimal fallow period.

In the SR-model increased real wage implies shorter fallow, because marginal net benefits are decreasing with increasing fallow length. Thus, the condition of marginal-costs-equal-marginal-benefits is restored if the fallow period is reduced, cf. (10). In the MR-model, the capital cost and the site value are reduced when z increases. The costs of increasing the fallow period is therefore reduced, and these more than outweigh the effect of increased z which determines the outcome in the SR-model (and, of course, still is present in the MR-model).

The MR-model presented here introduces two new aspects: Discounting (capital costs) and opportunity costs of land (site value). A very relevant question is which of these makes the conclusion in (28) switch compared to (16)? In Appendix 3 we show that the SR-model with discounting, and a sufficiently high discount rate, gives a result in line with (28). The MR-model with a zero discount rate is discussed below, and the conclusions from (28) and (29) are still valid under this assumption. Thus, introducing opportunity costs of land (i.e., a multi-rotation model) is sufficient, but not necessary, for the conclusion in (28). Introducing discounting is neither necessary nor sufficient to change the conclusion in (16), but it may if the discount rate is high, cf. Appendix 3.

4.4 Special case: Zero discount rate

A special case is obtained when the discount rate approaches zero. The first term on the RHS of (22) is zero. For the second term, by applying l'Hôpitals rule we get;

$$\lim_{\delta\to 0} \frac{\delta}{1-e^{-\delta m}} = \lim_{\delta\to 0} \frac{1}{me^{-\delta m}} = \frac{1}{m}$$

Thus, (22) becomes;

$$(22') \quad af_m - w(1+qb)g_m = \frac{af() - w(1+qb)(l+g)}{m} \Leftrightarrow r_m = \frac{r}{m}$$

This is the condition of maximum annual profit (or maximum sustainable rent - MSR), occurring when the marginal profit of increased fallow equals the annual profit. Note that the MSR is different from the maximum sustainable yield (MSY) concept, because MSR includes costs. The condition for MSY is in our case $f_m = \frac{f()}{m}$. The MSY, often advocated by foresters and environmentalists, is a special case when the discount rate is zero, and costs are ignored. Whereas the choice of discount rate is clearly debatable, ignoring costs could hardly be defended. If something is to be sustained over time, it should be rent, not yield.

Standard textbooks in resource economics, like Neher (1990: 72-73), distinguish between three solutions in forestry management: MSY, single rotation and multi-rotation, even though the MSY is a special case of the latter, as just shown. The derivation of the condition for MSR can be done in an alternative and simpler way. The optimizing problem can be written as to maximize the average annual rent (*AAR*) from a plot at a given distance (within the agricultural frontier);

(21') $Max_{m,l} \quad AAR = \max \frac{r(m,l)}{m}$

The FOC are;

(22'') $\frac{r_m m - r}{m^2} = 0 \Leftrightarrow r_m = \frac{r}{m}$

(23') $r_l = 0$

The first equation in (22'') exposes clearly the trade-off involved in the choice of fallow period in shifting cultivation. One the one hand, longer fallow period implies higher rent from the land when it is cropped. However, longer fallow implies that the land is cropped less frequently, which reduces the *AAR*. Equation (22'') states the optimal balance between these two considerations. The condition of marginal rent equals annual rent is illustrated in Figure 6 below.

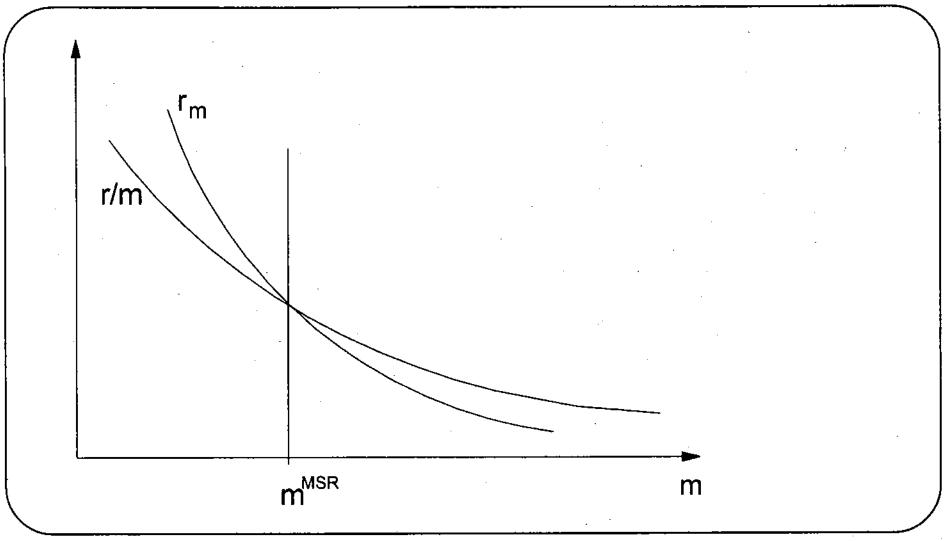

Figure 6. The Maximum Sustainable Rent (MSR) solution for m.

The expressions for the comparative statics in (26) are slightly different in the MSR case, but the results in (28) - (29) are still valid: Higher *z* leads to longer rotation and less labour input.

4.5 Environmental benefits

The main concern about tropical deforestation, of which a significant share is caused by shifting cultivators, is related to the environmental benefits produced by standing forests. These range from local ecological functions in terms of flood control, and soil protection, to global benefits like maintenance of biodiversity and storage of carbon in the biomass.

Environmental benefits (E) from standing forests can easily be included into the analysis, as first shown by Hartman (1976). E may be assumed to be a concave function of the age of forest: $E = E(m); E_m \geq 0; E_{mm} \leq 0$. The objective function for a plot with given distance then becomes;

$$(32) \quad Max\ NPV_{m,l} = \max \frac{1}{e^{m\delta}-1}\left[r(m,l;a,w(1+qb)) + e^{m\delta}\int_0^m e^{-y\delta}E(y)dy\right]$$

Necessary conditions for an interior solution are given in (23) and (33);

$$(33) \quad r_m + E(m) = \frac{\delta}{1-e^{-m\delta}}\left[r + \int_0^m e^{-y\delta}E(y)dy\right]$$

Compared to the standard Faustman equation in (22), the first terms on both sides are the same as before. The LHS gives the benefits of delaying clearing by one year, where the second term reflects the environmental benefits produced by a forest of age m. The RHS represents the costs related to delaying the cutting by one year, in terms of delayed future benefits. In addition to the production benefits (r), there are environmental benefits over the rotation cycle that should also be taken into account; this is reflected in the last term on the RHS.

Since we are adding a positive term on both sides of the Faustman equation, the impact on the rotation length of adding environmental benefits is not readily seen. In the single rotation problem, the RHS of (33) reduces to δr. In this case, the effect on the rotation period is clear: The optimal rotation is longer. Indeed, the optimal solution may be never to cut the forest, as elaborated by Strang (1983). Adding environmental benefits also implies that the agricultural frontier will be reduced.

In the multi-rotation case, Bowes and Krutilla (1985: 539) show that when environmental benefits are rising with stand age (as we have assumed) the rotation will also be longer. On the other hand, if the *marginal* benefit of increasing stand age is zero in the relevant area, the Faustman solution will still be the correct one, even if the *total* environmental benefits are high. Bowes and Krutilla (1985) also show that if the environmental benefits are large compared to the production benefits, a higher discount rate may lead to *longer* rotations.

4.6 Risk of loosing the land

Shifting cultivators face several types of risk: Yield, prices, biomass growth, and the risk of loosing land by fire, to external claimants, etc. The latter has been studied in the forest economics literature, and may be given an interesting application to shifting cultivation decision-making. Reed (1986: 184) shows that the effect of loosing the forest (by fire) is "the same as that of an increase in the discount rate by an amount equal to the average rate at which fire occurs".[26] This is an example of *risk discounting*. To get this result one has to assume an average risk

[26] See also Conrad and Clark (1987, chap. 5.3).

λ per unit time for loosing the land, as described by a homogenous Poisson process at rate λ. The relevant discount rate in this case would be $(\delta + \lambda)$, and with that adjustment the above results are still valid.

In the shifting cultivation setting, the most significant risk to a local decision maker would in many cases be that of loosing the land to an external claimant, for example a government sponsored plantation project. The effect of this, as seen from (30), is to shorten the fallow period, which corresponds to sound economic intuition: If there is a risk of loosing the forest, one should cut it relatively earlier to get the benefits, as waiting for another year entails the risk of loosing the land completely.

An empirically relevant modification would be that the probability of loosing the land is dependent on the age of the forest, in the way that the claims are weaker the older the forest is (the longer time since last cultivation), i.e., $\lambda = \lambda(m); \lambda_m > 0$. This would not change the results significantly. In both cases the logic of the Faustman result survives: In optimum the *expected* growth in stumpage value (taking into account the probability of loosing the land) should equal the capital cost and the site value.

4.7 Changing technology or prices over time

The above discussion has assumed that technology (production function), prices, the discount rate and all other parameters remain constant over time, which all are quite heroic assumptions.[27] Changing the assumptions that everything is known and constant over time adds more realism to the model, but complicates the analytics considerably. Two models are briefly referred to below.

One of the criticisms of the Faustman approach is that it may yield rotation periods shorter than the maximum sustainable yield (MSY) rotation. Thus, the Faustman solution is less likely than the MSY solution to maintain the productivity of forest land. Walter (1980) applies a production function where the number of previous rotations is included in the production function, not only rotation length. This would, in general, result in a solution with variable rotation length and a terminal date of exploitation of forestland. The optimality condition, however, resembles the Faustman equation and the same type of marginal calculation is involved.

Newman *et al.* (1985) discusses the case where the relative timber prices are increasing over time. If the rate of price change is less than the discount rate (which is a fair assumption), the optimality condition for the steady state is the Faustman equation, with the discount rate replaced by the interest rate *minus* the rate of price change (*real* interest rate). This is, however, based on some simplifying assumptions, particularly, no costs are included. If one includes costs, as done in

[27] "If the solution is to be simple, the assumptions must be heroic", Samuelson (1976: 470).

this paper, the direction of the result would remain the same, but one must use *net* prices, and the analytics would be more complex.

They question the usefulness of steady state results, and the main message in Newman *et al.* (1985) is to distinguish between the effects of *rising* relative price and the change in the price *level*. The second (level effect) implies a shorter rotation, as shown in (28). However, the effect of the first one points in the other direction, as seen from (30), remembering that a change in the rate of price change has an effect similar to a change in the discount rate.

> *"Increases in the rate of price change initially increases the optimal rotation. ... These two impacts eventually offset each other as the rotation lengths tend to a steady state. We would argue that the policy usefulness of the increase in the rate of price change is greater. That is, given discounting and the long production period in forestry, the positive impact on the initial rotation length is much more important than the steady-state rotation length or the elapsed time until the steady state" (Newman et al., 1985: 352).*

This illustrates how a more realistic description may change the results of the model. Note, however, that for shifting cultivation, where the rotation period is much shorter than in most natural forest timber production, their conclusion about the relative importance of the two impacts may not hold.

4.8 Summary & conclusions of the multi-rotation model

- Contrary to the single rotation (SR) model, fallow length is increasing in *z*, that is increasing with real wage and distance, and decreasing with travel efficiency and technological level.
- A lower discount rate leads to longer fallow periods. At the limit, when the discount rate approaches zero, we get the Maximum Sustainable Rent (MSR) case.
- In the same way as in the SR-model, labour inputs are decreasing in *z*. A lower discount rate leads to reduced labour inputs, but this conclusion is based on uncertain assumptions.
- The agricultural frontier is, as in the SR-model, negatively related to the level of *z*; lower *z* implies more deforestation.
- The frontier is, maybe surprisingly, independent of the level of the discount rate.
- Including environmental benefits of standing forests will, under realistic assumptions lead to longer fallow period and a contraction in the agricultural frontier.
- Adding the risk of loosing forestland has the same effect as an increase in the discount rate.

- Introducing rising output price over time yields two different effects on the fallow period: The effect of *rising* prices has the same effect as a lower discount rate (real interest rate), i.e., longer fallow, whereas the effects of a higher price *level* (lower z) is to shorten the fallow length.

5 Communal or private property

In theory, the multi-rotation social planner's solution(s) discussed above could be obtained under both a private or a communal property regime. Some authors, following Samuelson (1976), use private property rights and a competitive market to derive the Fautsman results above. Similarly, a communal management system, equivalent to a local social planner, would yield the outcome discussed above. The latter assumes that the local users can be treated as one decision-making unit, implying that internal co-ordination problems (like moral hazard) have been solved. If this is not the case, we may move in the direction of an open access solution, which is discussed in the next section.

Even though the formal modelling is the same, there may be some differences in the actual outcome. The first relates to the inclusion of public goods (or bads), like environmental benefits of standing forest. Generally, one may realistically assume that public goods are included only if they occur at the same geographical level as the management takes place. Thus communal management may include local environmental effects, like flood prevention, whereas global effects such as the carbon storage in standing forests may not be included in local management decisions. Similarly, private property may not include either of these environmental effects, but may take into account, for example, *on site* soil erosion.

A second reason as to why the adaptation may be different is due to differences in the discount rate. It is commonly argued that traditional societies have a low discount rate, for example, based on a cyclical perspective of life and a strong concern for future generations of their own society. At the extreme, with no discounting, the solution approaches the MSR-rule. On the other hand, private managers may apply discount rates much higher than the "socially correct" (however defined) discount rate. As shown above in (30), the higher the discount rate, the shorter rotation period. However, the agricultural frontier is independent of the discount rate in this model.

In other situations communal management may, for a variety of reasons, not be efficient in regulating individual farmers resource use. In such situations communal management may be moving towards an open access solution, which, as shown below, implies that the discount rate is *higher*.

6 Open access

6.1 The model

So far we have looked at situations with well defined and secure property rights. Now we move to the other extreme, that is a situation where no property rights exist neither before nor after clearing. In the next section we modify this and look at a situation where no rights exist before forest clearing, but property claims are established through forest conversion to agriculture (homesteading).

Under open access, given that there is competition for land, and land is acquired at no costs (except for labour inputs), the profit (land rent) is driven to zero. All rent is dissipated (Gordon, 1954). As in the previous models, all forest which can yield a non-negative rent, will be cleared. In equilibrium, farmers will be indifferent as to *where* they clear new land for swidden (as long as $b \leq b^{\max}$), and between farming and off-farm work (at wage *w).*

The model will then consist of the following equations (which are repeated here for convenience);

(7) $r = f(m^{\min}, l^{**}) - z[l^{**} + g(m^{\min})] = 0$

(5'') $f_l(m^{\min}, l^{**}) = z$

The endogenous variables are m and l, whereas z is the only exogenous variable. (7) defines the minimum fallow which gives a non-negative rent, and forest will be cleared as soon as $r \geq 0$. Note that (7) also gives the agricultural frontier (b^{max}) when $m = m^*$ as given in (5).

Compared to the Faustman-solution, open access implies shorter fallow periods (except at the frontier, where it is the same in both models). Thus, the labour inputs will also be higher inside the frontier in the open access model (but the latter is again crucially dependent on the assumption of $f_{ml} < 0$).

Neher (1990: 63) claims that the single rotation (SR) model "can apply on a 'frontier' where land is 'free' and the harvester intends to cut and then abandon the land and move on". He uses a SR-model where discounting is included, thus the optimal conditions in the SR-case are (11) and (10');

(10') $r_m = \delta r \Leftrightarrow \frac{r_m}{\delta} = r$

From this we see that a general statement about the SR model as a good description of a frontier economy with 'free' land is incorrect. However, the SR model solution approaches the open access solution as the discount rate goes towards infinity, as easily seen from (10'). As regards the multi-rotation (MR) case, (22) can be written as;

$$\frac{r_m}{\frac{\delta}{1-e^{-m\delta}}} = r$$

Applying l'Hôpitals rule we get;

$$\lim_{\delta\to\infty} \frac{\delta}{1-e^{-\delta m}} = \lim_{\delta\to\infty} \frac{e^{\delta m}}{m} = \infty$$

We see that also in the MR model the rent goes towards zero as the discount rate goes towards infinity. Thus, open access can be viewed as a special case in both social planner's models when the discount rate is infinite, and future values thereby neglected. In the open access case the reason is that the farmers have no claims to future benefits, in the social planner's case future benefits are discounted to zero. This is a general result in resource economics (e.g., Clark, 1990: 43), which also holds in our model.

6.2 Comparative statics

To see how farmers in an open access situation respond to changes in z, including how fallow and labour inputs vary with distance, we differentiate the model with respect to its variables;

(34) $[f_m - zg_m]dm + [f_l - z]dl = [l + g]dz$

(35) $f_{lm}dm + f_{ll}dl = dz$

From (5) we immediately see that the second [ ] in (34) is zero, so we can solve the system recursively, first for m in (33), and then use this to solve for l in (35). Note that the first [ ] in (34) is positive, as it is evaluated in the open access situation, and not the optimal fallow in the single rotation problem in (10) where [ ] is zero. Rewriting (34) and (35) gives;

(36) $\frac{dm}{dz} = \frac{l+g}{f_m - zg_m} > 0$

(37) $\frac{dl}{dz} = \frac{1}{f_{ll}} - \frac{f_{lm}}{f_{ll}}\frac{dm}{dz} = \frac{1}{f_{ll}} - \frac{f_{lm}}{f_{ll}}\frac{(l+g)}{(f_m - zg_m)} < 0$

The fallow period is thus increasing in z, for example, higher real wages will increase the length of fallow for a given distance. Further, the more distant fields have a longer fallow period and higher yield per clearing, which is necessary to compensate for the distance costs. To get the shape of the relationship between fallow period and distance, (36) can be differentiated further (here with respect to b, not z).

$$(38) \quad \frac{\partial^2 m}{\partial b^2} = \frac{(\frac{w}{a}q)^2(l+g)g_m}{(f_m - zg_m)^2} > 0$$

The fallow will increase with distance from the village, and at an increasing rate, as illustrated in Figure 7 below. At $b=0$ we have $m = m^{min}$, whereas $m = m^*$ at $b = b^{max}$ (8). The reason for the positive relationship between length of fallow and distance is different than for the Faustman solution: Under open access the condition is that rent everywhere should be zero. This means that land with low distance costs will be cut earlier. We get the convex relationship in the figure due to the concave relationship between fallow and yield.

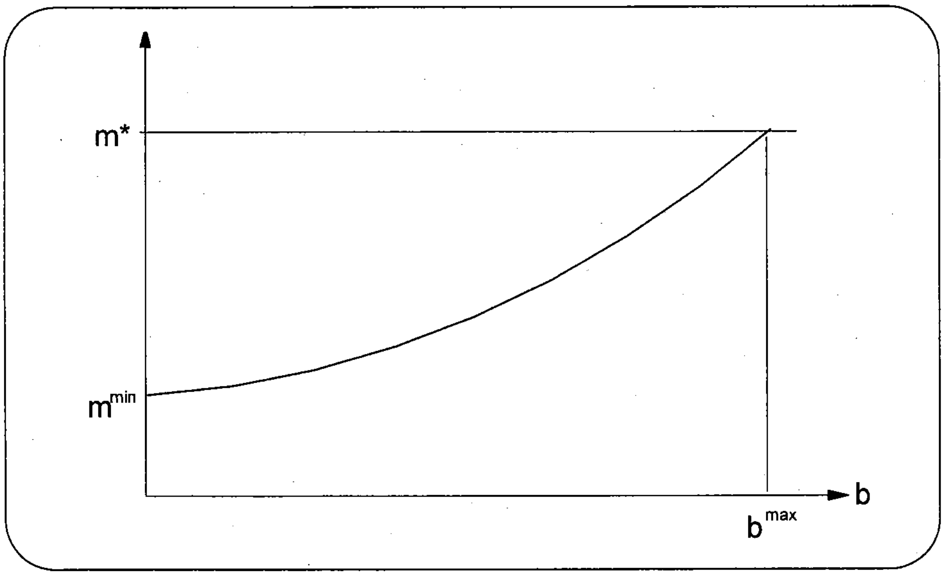

Figure 7. The relationship between fallow period (m) and distance (b) under an open access regime.

From (37) we see that an increase in z would lower the labour inputs for two reasons: The direct effect is that labour has become more expensive. As an indirect effect, the increased fallow length will, as we have assumed, lower the marginal productivity of labour.

As regards the agricultural frontier, it will be the same as under the social planner solution. The frontier is defined as the maximum distance where the rent is non-negative, and this would be the same in all cases. Consequently, the effect on b^{max} following an increase in z will also be the same as in the previous cases.

6.3 *Adjustment is costly and takes time*

The discussion above is based on a comparison between different long term equilibria or steady states. To see what happens in "real life" after an exogenous shock, consider the case when there is a sudden jump in the technological level (z down), and adjustment to a new equilibrium is costly and takes time (for example limited mobility). Immediately following the exogenous shock, there are opportunities to capture some rent because land is available with a fallow-distance combination that yields a positive profit. This is illustrated in Figure 8. In particular, the opportunities for rent capturing are highest close to the village, where the distance costs are lowest. Thus, the short term effect will be a concentration of farming close to the village where the rent is highest, whereas the long term effect is an expansion of the agricultural frontier.

This phenomenon was observed in the shifting cultivation district of Seberida, Sumatra, Indonesia (Angelsen, 1994). After the mid-1980s *z* was lowered due to lower transport costs (*q*), higher agricultural price (rubber) and lower opportunity cost of labour (lower *w*). Farmers' response in the period up to 1989 was to take land closer to the village, with a shortening of the fallow period. The share of households involved in shifting cultivation increased significantly and rubber planting increased. Since 1989, when less land was available close to the village, the swiddening has moved further away from the village, clearing older forest and increasingly primary forest as well.

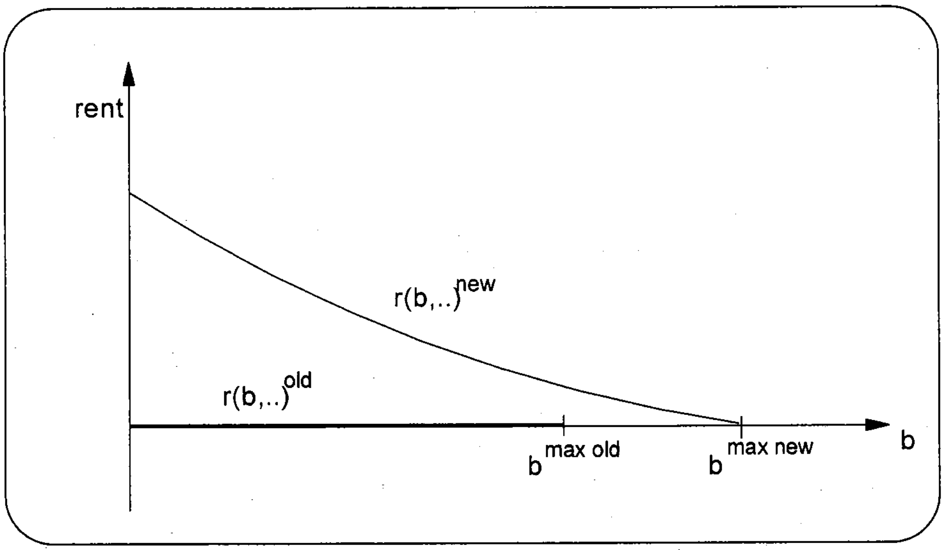

Figure 8. Land rent immediately after an exogenous shock that lowers z (e.g., technological progress).

6.4 Summary & conclusions of the open access model

- The fallow period increases, and at an increasing rate, with distance from the village under an open access regime.
- All potential land rent is 'competed away' as forest will be cleared for shifting cultivation as soon as the combination 'fallow-distance' is such that it yields a non-negative rent.

- Somewhat surprisingly, the agricultural frontier (and deforestation) will be the same under open access as in the two social planner's solutions (provided all other variables are the same).
- The agricultural frontier will, as in the previous cases, be moved further away by technical progress (a), a lower real wage rate (w), and a decline in the travel efficiency factor (q), for example by new/improved roads.
- The intensity of production will increase (the fallow period decrease) with technical progress (a), shorter distance (b), improved travel efficiency (lower q) and lower real wage rate (w).
- If we introduce time and costly adjustments to a new equilibrium, the short and long term effects following, for example, technological progress may be quite different: In the short term land close to the village is "up for grasp", whereas the long term effect is an expansion of the agricultural frontier.

7 Homesteading: Private property rights established by clearing

7.1 The model

A common feature of shifting cultivation systems is that forest clearing gives the farmer some (usufructuary) rights to the land, particularly if perennials are planted or the land is 'improved' in some other way. The protection of these rights in customary and national law varies considerably, and we shall later include unsecured property rights in the model. People involved in such agricultural practices are sometimes referred to as 'colonists' or 'squatters', but both farmers having lived in the area for a long period as well as newcomers may be engaged. This is indeed the case in the Seberida district, Sumatra, where most of the expansion is due to local farmers, but transmigrants (migrants from Java) are increasingly taking up the shifting cultivation practice.

This regime has a parallel to homesteading in the United States, officially introduced with the Homestead Act of 1862 and ending in 1934 (Allen, 1991; Anderson and Hill, 1990). A large share of the literature on homesteading has focused on the negative aspects of such a regime, because it causes farmers to rush to the land in order to gain property rights. In this process, all positive future rents and the potential gains from agricultural expansion are dissipated.

A homesteading regime implies that forestland is transferred from being an open access resource to private property. Compared to the previous open access model, such a regime for obtaining property rights makes it necessary to include some new elements in the model: Farmers will not only look at the immediate benefits from one cultivation, but also the future gains (net present value - NPV). This makes it necessary to include another aspect, that is farmers' expectations about the factors that determine future land rent. For simplicity, we assume risk neutral agents, i.e. the expected NPV is maximized. We retain the zero-profit condition, except that the relevant profit now is the NPV of future land rent.[28] We assume further that the forest is managed optimally, in the way described by the Faustman rule, after the initial clearing, when private property rights have been established. For simplicity we set $a = 1$. The model for the initial clearing consists of two equations, the zero-profit equilibrium in (39), and the labour input equation in (5''').

$$(39) \quad NPV_k = f(m^1, l^1) - z[l^1 + g(m^1)] + NPV^{MR}(z^e) = 0$$

$$(5''') \quad f_l(m^1, l^1) = z$$

The net present value at time k is the land rent from the first clearing, plus the NPV of future land rent after the initial clearing. The superscript *MR* refers to the multi-rotation (Faustman) solution, *1* refers to first clearing, whereas *e* indicates expected values.[29]

For land inside the border of cultivation we have $NPV^{MR} > 0$. Thus, according to (39), the profit from the first clearing will be negative. Farmers are willing to accept a loss as the NPV of consequent clearings is positive, and getting rights to this benefit stream (that is property rights) outweigh their initial loss. Forest cleared to establish property rights will be younger than for the subsequent rotations. Thus, compared to the situation of open access where clearing does not give property rights, the initial rotation will be shorter, whereas the later will be longer inside the border.

As regards the agricultural frontier, we must distinguish between two cases. In the first one $z^e = z^{p(resent)}$, i.e., farmers expect the present effective real wage to remain the same in the future. In this case the agricultural frontier will be the same in both open access regimes. At the frontier, NPV^{MR} is zero, thus the profit from the first clearing must also be zero. And we have argued earlier that we get the same frontier in the Faustman and 'pure' open access case, thus the agricultural frontier remains the same also in this case.

As a second case we look at the situation when $z^e < z^p$, i.e., farmers expect the effective real wage to decline due to, say, technological progress or lower transport costs. As the Faustman solution assumes all parameters to be constant, we have to think of this as a one-time drop in z after the first clearing. Consider a situation at the agricultural frontier as given in the first case above, where $z^e = z^p$. If now z^e drops, the NPV of future land rent at the frontier becomes positive. Therefore, it will be profitable to expand the frontier, even if this gives a loss the first clearing.

[28] One may argue that the NPV which should always form the basis for the decision, but that in the 'pure' open access the NPV is equal the land rent, as the farmers have no rights to future land rent.

[29] Note that NPV^{MR} is not discounted as it is assessed for bare forest land (i.e., at the time of clearing).

In this case the frontier under homesteading will be moved further away from the village compared to the pure open access case. This is shown more formally below.

The two open access regimes are compared in Figure 9 below: The agricultural frontier will be moved further away when we have a regime that allocates property rights to the settlers, and $z^e < z^p$ (case 2).

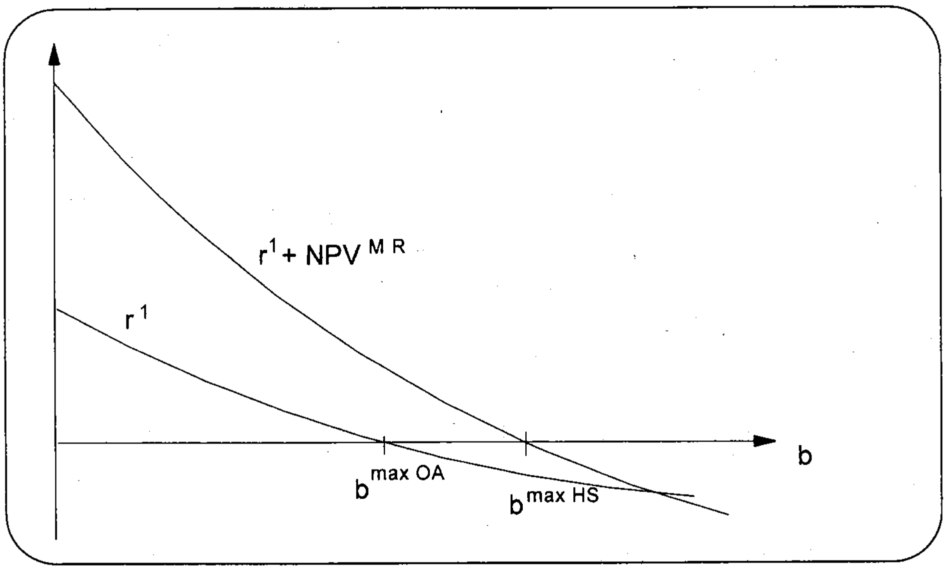

Figure 9. Agricultural frontier under two types of open access regimes.

7.2 Comparative statics

To see what happens if z^e declines (z^p remains constant) we differentiate the model, (5''') and (39). We focus on the effect of fallow length and labour input for the first clearing, and the agricultural frontier. The effect on m and l *after* property rights have already been secured, has been discussed under the multi-rotation model.

(40) $f_{lm}dm^1 + f_{ll}dl^1 = 0$

(41) $(f_m - z^p g_m)dm^1 + \frac{dNPV^{MR}}{dz^e}dz^e = 0$

We see that the solution is still recursive, i.e., we can first determine the changes in m^l from (41) and then use this to determine l^l from (40). Rearranging (41), and using it in (40) give;

(42) $\frac{dm^1}{dz^e} = -\frac{\frac{dNPV^{MR}}{dz^e}}{f_m - z^p g_m} > 0$

(43) $\frac{dl^1}{dz^e} = -\frac{f_{lm}}{f_{ll}}\frac{dm^1}{dz^e} < 0$

The interpretation of these results is straightforward. Lower expected effective real wage means higher expected land rent in the future. Thus, farmers will clear younger forest, with larger loss from the initial clearing, in order to get rights to the land and the higher future land rent. Labour inputs will increase with lower z^e because shorter fallow means higher marginal productivity of labour, but this conclusion again rests on thin empirical evidence.

What is the effect on the agricultural frontier? Assuming again that we are starting in a situation where $z^e = z^p$, and remembering that both for the initial and the following clearings the fallow period at the frontier would be m^* (5), differentiation of (39) gives;[30]

$$(44) \quad -w^1 q^1 [l^1 + g(m^1)] db^{\max} + \frac{dNPV^{MR}}{dz^e} dz^e - \frac{1}{e^{m^{MR}\delta} - 1} [l^{MR} + g(m^{MR})] w^e q^e db^{\max} = 0$$

$$(45) \quad \frac{db^{\max}}{dz^e} = \frac{\frac{dNPV^{MR}}{dz^e}}{w^1 q^1 [l^1 + g(m^1)] + \frac{1}{e^{m^{MR}\delta} - 1} [l^{MR} + g(m^{MR})] w^e q^e} < 0$$

The result of (45) has a similar interpretation as the one above: Lower expected effective real wage will lead to an expansion of the agricultural frontier. An increase in the expected NPV means that farmers would be willing to move further away and accept higher losses to get the higher future land rent.

The effect of a change in the discount rate can be analyzed in a similar way, and we get;

$$(46) \quad \frac{db^{\max}}{d\delta} = \frac{\frac{dNPV^{MR}}{d\delta}}{w^1 q^1 [l^1 + g(m^1)] + \frac{1}{e^{m^{MR}\delta} - 1} [l^{MR} + g(m^{MR})] w^e q^e} < 0$$

The intuition is similar as for (45). A higher discount rate would lower the NPV of future land rent. Thus, the initial age of the forest cleared will be higher, and the agricultural frontier closer to the centre. Thus, somewhat surprisingly, a higher discount rate implies less deforestation because the positive future land rent is given less weight in farmers' decision. This contradicts conventional wisdom which holds that lower discount rates would help preserve the environment. This is not true in our model under the homesteading property rights regime.

7.3 Two possible scenarios

This far the discussion has compared different long term equilibria. Adjustment will take time, both due to the long rotation periods involved, and because there are adjustment costs. Consider a situation where $z^e = z^p$, that is farmers plan as if the effective real wage will remain the same in the future. Property rights have been established for all land up to the agricultural frontier. Then there is an exogenous

[30] Note that the effect on (41) of a marginal change in m or l is zero.

decrease in z^e. This implies that the expected NPV for some land outside the old border of cultivation is "up for grasp". Farmers will compete among themselves for the rights to the land, and we have a "race for property rights" or uncaptured land rent.

Another related scenario would be where there exists some form of communal management of the forest. z^e is expected to decrease in the future. However, since communal rights to all forestland are already established, there is no need to clear land to secure property rights. New land is not cleared before it yields a positive (or non-negative) rent from the first clearing. Consider now the effect of the forest being opened to outsiders for competition, i.e. we move from a regime of communal management to open access where clearing gives property rights. In this scenario, even the expectation of such a change alone, could cause an expansion of the agricultural frontier.

The homesteading case and both scenarios draw the attention to the importance of *expectations*, and how these are formed. In Angelsen (1994) we argue that, in our study area in Sumatra, state sponsored land claims (primarily transmigration, logging, and plantation projects) have been important in initiating a land race. Eventually, however, the main driving force is internal mechanisms in terms of increased competition among the farmers, expressed by increased forest clearing and rubber planting. Such land races may therefore be self-reinforcing through their impact on farmers expectations.

7.4 Uncertainty about the future rights

Most 'real life' situations would be in-between the two extreme cases discussed above; pure open access and homesteading with certain property rights. Generally, uncertainty about future rights would give a solution between the two extremes.

This can be discussed analytically in at least two ways. One possibility is just to add a parameter, β, before NPV^{MR} in (39), with $0 \leq \beta \leq 1$. $\beta = 1$ represents the homesteading case just discussed, whereas $\beta = 0$ is the open access case where clearing gives no property rights. This assumes a rather specific structure on the nature of the risk of loosing the land.

Another possibility will be to combine this with the Reed (1986) approach, as discussed under the multi-rotation social planner problem. We argued that the risk of loosing land has the same effect as a higher discount rate, which is also a key result in Mendelsohn (1994). And, as shown in (46) the effect of a higher discount rate is a contraction of the agricultural frontier.

Thus, in both approaches introducing unsecured future rights would *reduce* the agricultural expansion compared to the case of secure property rights. We are moving from a homesteading solution with secure rights towards an open access solution, which as shown, implies less forest clearing.

7.5 Alternative assumption: Initial situation is old growth forest

The discussion of homesteading is, in line with the tradition of the Faustman literature, based on the assumption that one star from a situation with bare land, i.e., all forest is of age *0* initially. An alternative, and possible also more realistic formulation would be that initial situation is one with old growth (climax or primary) forest. This would actually simplify the analysis, as shown below. The homesteading model would then be;

$$(47)\qquad NPV_k = f(m^{OG}, l^{OG}) - w(1+qb)[l^{OG} + g(m^{OG})] + NPV^{MR}(z^e) = 0$$

$$(48)\qquad f_l(m^{OG}, l^{OG}) = w(1+qb)$$

The choice variables for the initial clearing and cultivation are the labour input in old growth (*OG*) forest, and the agricultural frontier. The initial fallow period (m^{OG}) is given and not a choice variable here. As before, the farmer is assumed to follow a Faustman optimization after the initial clearing.

Further, we define the rent from cultivation of old growth forest (r^{OG});

$$(49)\qquad r^{OG} = f(m^{OG}, l^{OG}) - w(1+qb)[l^{OG} + g(m^{OG})]$$

It may well be that $r^{OG} < r^*$, as given in (6). This is illustrated in Figure 10 below, cf. also Figure 2 above. Boserup (1965: 31), Dvorak (1992) and others report that swidden farmers in general have a preference for secondary forest due to the high costs of clearing old growth forests. This is in line with the assumptions made for the *f(m,l)* and *g(m)* functions. Figure 10 shows that rent as a function of forest age reaches its maximum, r^*, at m^*, and then declines until the forest reaches its climax at m^{OG}, a pattern which is in line with empirical observations. After this age the forest is in a steady state, and the rent does not change.

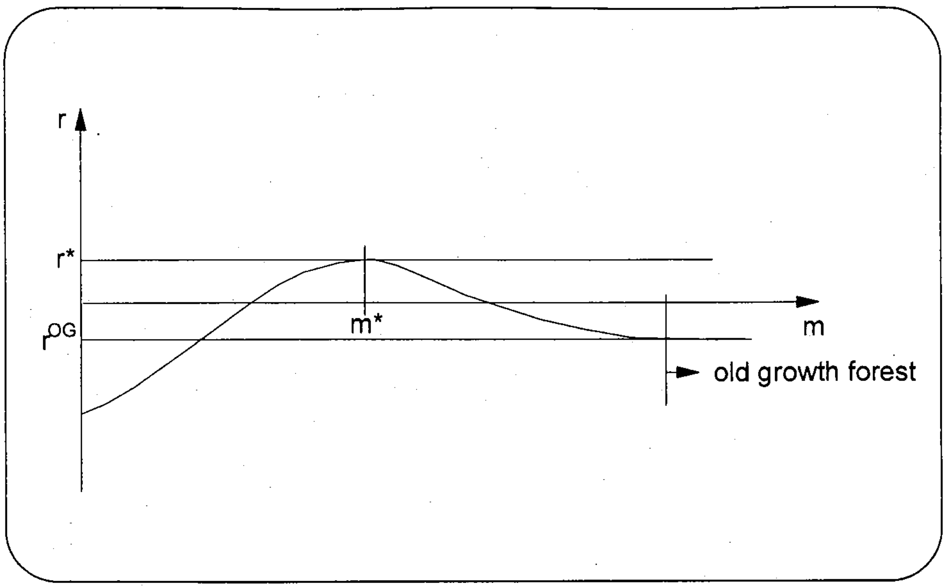

Figure 10. Rent in old growth forest (r^{OG}) and maximum rent (r^*).

Farmers clearing old growth forest may have to take a loss for the initial clearing and cultivation because of the high clearing costs, whereas the following cultivations would yield positive rents. They are willing to take this loss in order to get the rights to future rent, as in the case of bare land initially.

However, in contrast to the result when we assume to start from bare land, we note a situation with an initial loss may be the case even if we have $z^p = z^e$. In equilibrium, all rent is captured immediately. Thus, all farmers clearing forest at the frontier have to accept a loss for the first clearing and cultivation. If now the expected effective real wage (z^e) declines, farmers would expand the frontier further, and be willing to take an even larger loss during the initial clearing to get the property rights. Thus, the conclusions and the comparative statics results in this version of the homesteading model remain the same as when assuming to start from bare land.

7.6 Summary & conclusions of the homesteading model

- Compared to the 'pure' open access model, the model where property rights are obtained through forest clearing implies shorter initial fallow period, whereas the subsequent fallows are longer.
- If the effective real wage is expected to be lower in the future than it is presently, then this property regime would lead to more agricultural expansion and deforestation.
- If we assume that the initial situation is old growth forest, then the homesteading regime will give more deforestation than the open access

solution, even in the case when the effective real wage is expected to remain the same in the future.

- In addition to the factors identified under the 'pure' open access model, some additional factors can contribute to an expansion of the agricultural frontier: A breakdown of communal management towards an open access regime where clearing gives rights, lower expected values of *z*, and a lower discount rate!
- Compared to the socially optimal solution, homesteading gives a too early conversion of forestland to agriculture, and induces a welfare loss (in addition to possible welfare losses due to negative environmental effects that are not included in the decision-making).

8 Comparison of the different property rights regimes

This paper has combined the Faustman rotation approach and the von Thünen spatial approach in a model for a small, open economy, i.e., a model with a perfect labour market and exogenous real wage. We have looked at four different solutions to this model, corresponding to different assumptions about the property rights regime. A comparison of the variables and the effects of exogenous changes is given in Tables 1 - 3 below.

	Social planner: Single rotation (SR)	Social planner: Multi-rotation (MR)	Open Access (OA)	Homesteading (HS) (1. rotation)
Fallow period	m^{SR}	$> m^{MR}$	$> m^{OA}$	$> m^{HS}$
Labour input	l^{SR}	$< l^{MR}$	$< l^{OA}$	$< l^{HS}$
Agr.frontier	$b^{max\ SR}$	$= b^{max\ MR}$	$= b^{max\ OA}$	$< b^{max\ HS}$

Table 1. Comparison of the level of endogenous variables under different regimes. (Under the HS case, the values refer to the initial clearing (for the subsequent clearings the MR-solution is followed), and we assume that the expected z is lower than the present one.)

The fallow period for a particular *z* will be longest in the single rotation problem, where the profit from one clearing is maximized and there is no cost of delaying the clearing. The shortest fallow is obtained in the homesteading regime, as farmers are willing to accept a negative profit during the first clearing(s) in order to get property rights to future positive land rents.

The margin of cultivation will be the same in all cases, except the last one when *z* is expected to decline in the future. This is somewhat surprising but the reason is straightforward. The agricultural frontier is the maximum distance possible without getting a negative rent. In the first three models this means that the rent from one clearing must be maximized, and set equal to zero at the frontier.

In the last model (homesteading), a negative rent is accepted for the first clearing, given that the rent is expected to increase in the future. Thus, the frontier is moved further away. This case is probably the most realistic description of large areas of tropical forest. It illustrates the importance of property rights, and how an ill-designed regime may produce perverse environmental outcomes.

The conclusion above that the agricultural frontier is the same for the first three models is modified if we include environmental benefits in the model. Obviously, an open access solution - even if it does not give the farmer property rights - will lead to more deforestation than the social planner's solution. The same is true for a private property regime.

The effects of an increase in the effective real wage (z) is shown in Table 2. In all cases, except the single rotation model, the fallow period will increase. In the multi-rotation model higher z implies lower opportunity costs of delaying clearing, and therefore longer fallow. In the two open access models higher z reduces the relative profitability, and an increase in fallow period is necessary to retain non-negative profit. Labour inputs decrease in all cases, as is to be expected.

	Social planner: Single rotation (SR)	Social planner: Multi-rotation (MR)	Open access (OA)	Homesteading (HS)
Fallow period (m)	decrease	increase	increase	increase
Labour input (l)	decrease	decrease	decrease	decrease
Agr.frontier (b^{max})	decrease	decrease	decrease	decrease
Profit (NPV or r)	decrease	decrease	no effect (= 0 by assumption)	no effect (= 0 by assumption)

Table 2. The effects of an increase in the effective real wage (z) on endogenous variables.

An increase in z will in all models cause a contraction of the agricultural frontier. Any policy that increases the relative profitability of shifting cultivation will lead to an expansion and increased deforestation. This seems to be a robust conclusion, not dependent on the actual property rights regime.

	Social planner: Single rotation (SR)	Social planner: Multi-rotation (MR)	Open access (OA)	Homesteading (HS)
Fallow period (m)	na	decrease	na	1. clearing: Increase Later clearings: Decrease
Labour input (l)	na	increase	na	1. clearing: Decrease Later clearings: Increase
Agr.frontier (b^{max})	na	no effect	na	decrease
Profit (*NPV* or r)	na	decrease	na	no effect (= 0 by assumption)

Table 3. The effects of an increase in the discount rate on endogenous variables. (See also comments on Table 1.)

Finally, the effects of an increase in the discount rate is summarized in Table 3. In the multi-rotation model the fallow period declines because the opportunity costs of delaying the clearing and harvest is increased. The labour input increases, whereas the discount rate has no effect on the agricultural frontier. In the homesteading regime the effect of an increase in the discount rate is to put less emphasis on the positive land rent in the future. Thus, the age of the forest cleared initially will increase, whereas later fallows will follow the MR-model, i.e., decrease. The reduced weight given to future positive rents also implies that the agricultural frontier will decrease. Because of the perverse incentives under homesteading we get that lowering the discount rate yields more forest clearing.

9 Concluding remarks

The main line of argument throughout the paper is that the intensity of cultivation (inverse of fallow period) as well as the agricultural frontier is determined by the relative profitability of shifting cultivation, as captured in a single variable - the effective real wage (z). The main force towards intensification in terms of shorter fallow periods and an expansion of the system is lower z, which is in turn determined by five variables: Agricultural price, nominal wage, technological level, transport costs, and distance. Policies affecting these factors can be used to influence intensity of production and agricultural expansion (deforestation).

The open economy approach employed in this paper should be contrasted with a subsistence approach, as discussed in the introduction. The underlying mechanisms in the two models are very different. In the open economy, relative profitability is the key word; in the subsistence model the subsistence demand from the population is the driving force.

A major aim for the clear distinction between these two approaches is to clarify some confusion that often arises in the debate on which factors affect agricultural expansion and deforestation. Sometimes the underlying assumptions are not clearly spelled out, and they turn out to be more significant than it appears. Some important examples of the different effects of exogenous changes in the two models are:

1. *Population growth.* Population growth has no effect in the open economy model, as the size of the agricultural sector (and its expansion into primary forest) is determined by its relative profitability.[31] In a subsistence model population growth is a critical variable in determining variables like forest clearing. FAO (1992: 11), for example, uses population as the only explanatory variable in their deforestation model. Time series data on the increased primary forest clearing in Seberida, Sumatra show that population growth contributed directly only 13 pct. to the total increase (Angelsen, 1994). The indirect effects, however, may be larger.
2. *Technological progress.* In an open economy model technological progress will increase the profitability and therefore expand the agricultural sector. In a subsistence model technological progress implies that the subsistence requirement can be met by cultivating less land.
3. *Increased risk.* In the open economy case increased risk makes risk-averse farmers reduce the scale of the risky activity, i.e., farming. This hypothesis is supported by, among others, Elnagheeb and Bromley (1994) in a study from Sudan. In the subsistence case, on the other hand, increased risk implies a larger area under cultivation as risk averse farmers would aim to be on the safe side of the subsistence requirement.[32]

A major lesson from the discussion is the need to focus on the structural properties of the economy, particularly on how the labour market works. The role of property rights has been extensively focused on in the debate on natural resource management in tropical agriculture, and rightly so. This paper shows that for the effect of exogenous changes on shifting cultivation intensification and expansion, the labour market assumptions may, in fact, be more important than the property rights assumptions. This may suggest a change in the focus of the empirical research in this area.

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31 Indirectly, population growth may affect variables like the real wage rate.

32 One possible behavioural assumption for subsistence farming under risk is that farmers minimize the probability of yield below a subsistence requirement, or that they minimize labour input, given a predetermined acceptable probability for output falling below subsistence (safety first models). See for example Roumasset (1977) for a more detailed discussion.

Appendix 1

We want to show that the second order conditions (SOC) to the problem in section 3 ensures that at least one of the expressions in (16) and (17) are negative.

First, we see that it is consistent with the SOC, $D = a_{11}a_{22} - a_{12}a_{21} > 0$, that both (16) and (17) are negative, i.e., $a_{22} < a_{12}$ *and* $a_{11} < a_{21}$. Remembering that $a_{ij} < 0;\ i,j = 1,2$, we see that the two equations being negative implies that $D > 0$.

Second, we must show that if either (16) or (17) is positive, the other one must be negative. Assume (16) is non-negative, i.e., $a_{22} \geq a_{12}$. We want to prove by contradiction that then (17) is negative. Assume the opposite to be the case, i.e., $a_{11} > a_{21}$. In a similar manner as above, it then follows that $D < 0$, which is not true. Thus (17) cannot be positive. Because of the symmetry, the same argument holds if (17) is assumed to be positive; then (16) must be negative.

Appendix 2

In this appendix we prove that b_{12} in (26) is negative, i.e., $b_{12} = (1 - e^{-\delta m})g_m - \delta(l + g) < 0$. We recall that $g()$ is concave; $g(), g_m > 0$. l and m are endogenous variables; $l, m > 0$. We also have a positive discount rate; $\delta > 0$.

$b_{12} < 0$ is equivalent to;

$$\theta = \frac{(l+g)}{g_m}\frac{\delta}{(1-e^{-\delta m})} = \mu\tau > 1;\ \mu = \frac{(l+g)}{g_m};\ \tau = \frac{\delta}{(1-e^{-\delta m})}$$

We always have that $\mu = \frac{l+g}{g_m} > m$; concavity ensures that $\frac{g}{m} \geq g_m$, and adding l $(l > 0)$ gives that the expression is strictly greater than m.

Then we must show that $\tau = \frac{\delta}{(1-e^{-\delta m})} > \frac{1}{m}$, which consists of two steps. First, we explore what happens when the discount rate goes to zero, using l'Hôpitals rule;

$$\lim_{\delta\to 0} \tau = \lim_{\delta\to 0} \frac{\delta}{1-e^{-\delta m}} = \lim_{\delta\to 0} \frac{1}{me^{-\delta m}} = \frac{1}{m}$$

Next, we take the derivative of τ with respect to the discount rate, and show that this is positive;

$$\frac{\partial\tau}{\partial\delta} = \frac{1-e^{-\delta m}-\delta m e^{-\delta m}}{(1-e^{-\delta m})^2}$$

This expression is positive iff;

$$\frac{1-e^{-\delta m}}{\delta m e^{-\delta m}} = \frac{e^{\delta m}-1}{\delta m} > 1$$

This is true iff $\phi(\delta) = e^{\delta m} - \delta m > 1$, which holds for any $\delta > 0$, as we have;

$$\phi(0) = 1; \frac{\partial\phi}{\partial\delta} = m(e^{\delta m} - 1) > 0 \quad if \quad \delta > 0$$

Thus, we have shown that τ approaches $\frac{1}{m}$ as the discount rate approaches zero, and that τ increases as the discount rate increases. Thus, we always have $\tau > \frac{1}{m}$, which combined with $\mu > m$ gives that $\theta = \mu\tau > 1$, i.e., b_{12} will always be negative.

Appendix 3

This appendix gives a very brief presentation of the single rotation model *with* discounting. We focus only on the determination of m and l as b^{max} will be as in the case without discounting. The objective is to maximize the discounted rent (DR) from a plot at a particular distance within the frontier;

(A1) $$Max\ DR_{m,l} = \max\ e^{-m\delta}\{af(m,l) - w(1+qb)[l+g(m)]\} = \max e^{-m\delta} r(m,l)$$

The FOC are;

(A2) $$\frac{\partial DR}{\partial m} = e^{-m\delta}[af_m - w(1+qb)g_m] - \delta e^{-m\delta}\{af(m,l) - w(1+qb)[l+g(m)]\} = 0$$

$$\Leftrightarrow\ r_m - \delta r = 0$$

(A3) $$\frac{\partial DR}{\partial l} = e^{-m\delta}[af_l - w(1+qb)] = 0\ \Leftrightarrow\ r_l = 0$$

(A2) is a well known formula in resource economics (and capital theory); the relative growth rate should equal the interest (or discount) rate: $\frac{r_m}{r} = \delta$. (A3) is the same as in the SR-model without discounting.

Differentiation of (A2) and (A3), and simplifying the notation by setting $a = 1$, and using that $z = w(1\text{-}qb)$, yields;

(A4) $$(r_{mm} - \delta r_m)dm + (r_{ml} - \delta r_l)dl = rd\delta + [g_m - \delta(l+g)]dz$$

$$or\quad a_{11}dm + a_{12}dl = b_{11}d\delta + b_{12}dz$$

(A5) $$r_{lm}dm + r_{ll}dl = dz$$

$$a_{21}dm + a_{22}dl = dz\ (b_{21} = 0; b_{22} = 1)$$

The second order conditions are assumed to be satisfied, particularly $D = a_{11}a_{22} - a_{12}a_{21} > 0$. All $a_{ij} < 0$, whereas the sign of b_{12} is ambiguous; for low discount rates b_{12} is positive, whereas it is negative for high ones (see the elaboration below). The effect on m and l of changes in the exogenous variables can be written as;

(A6) $$\frac{dm}{dz} = \frac{a_{22}b_{12} - a_{12}b_{22}}{D}\ ?$$

(A7) $$\frac{dl}{dz} = \frac{a_{11}b_{22} - a_{21}b_{12}}{D} < 0$$

(A9) $$\frac{dm}{d\delta} = \frac{a_{22}b_{11}}{D} < 0$$

(A9) $\frac{dl}{d\delta} = \frac{-a_{21}b_{11}}{D} > 0$

The result in (A7) makes the plausible assumption that the direct effect of higher labour costs (z) dominates over a possibly opposite indirect effects via changes in m. The result of (A8) corresponds with intuition; a higher discount rate yields shorter fallow periods because the costs of delaying the clearing (capital costs) are higher. And, shorter fallow means higher labour inputs because the marginal productivity of labour is assumed to increase (A9).

Due to the uncertainty of b_{12} the result in (A6) is uncertain. Higher z implies lower m, as in the single rotation model without discounting, if the nominator is negative, that is;

(A10) $a_{22}b_{12} - a_{12}b_{22} = f_{ll}g_m - f_{ll}\delta(l+g) - f_{ml} < 0 \Leftrightarrow \delta < \frac{f_{ll}g_m - f_{ml}}{f_{ll}(l+g)} = \delta^*$

Thus, for discount rates lower than δ^*, increased z leads to shorter fallow period. In section 3.2 we explained why higher z leads to lower m in the single rotation model without discounting. Here this result changes if the discount rate is sufficiently high. Discounting makes delaying clearance and production costly, and the higher discount rate, the higher is this *capital cost*. Higher z means that rent is reduced, therefore the capital cost is also reduced. Thus, the magnitude of the effect on rent and capital cost of increased z is positively related to the level of the discount rate. If the discount rate is high, this effect will dominate, and we change the result in (16) in section 3.2 by introducing discounting.

It would be of interest to find out something more about the critical value of the discount rate, as found in (A10) above. One may argue realistically that f_{ml} is relatively insignificant compared to f_{ll}, and we assume that $f_{ml} = 0$ (i.e., $a_{12} = 0$). Then we have;

(A11) $\delta^* = \frac{g_m}{l+g} < \frac{1}{m}$

The inequality follows from the concavity of $g()$, and that $l > 0$. This implies that if, as an example, $m = 20$, then the critical value of the discount rate would be *below* 5 percent, i.e., a 5 percent discount rate would change the result of the analysis compared to the case without discounting. Similarly, with a fallow period of 10 years the critical discount rate would be below 10 percent. How much below would depend on the size of l (relative to g), and the particular form of the $g()$-function. One conclusion is, however, that the sign in (A6) can be either positive or negative under realistic assumptions about the variables and functional form.

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