A selected survey of traditional and evolutionary game theory

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CMI Working Papers

Indexing terms

Game theory
Literature reviews

1. Introduction[1]

"A game is being played whenever people interact with each other", Binmore (1992, p.3). This statement implies that virtually every sphere of human endeavour and activity can be formulated as a game. If this is truly the case, then clearly game theory is worth exploring. Binmore (1992) was, however, quick to point out that, this does not mean that game theorists are the people to ask for answers for all the world's problems. The reason for this word of caution is that game theory, in its current state (what we refer to as traditional game theory in this paper), is mostly about what happens when people interact in a *rational*[2] manner. The big question is, do "real" people behave rationally in their day to day interactions? Traditional game theory (TGT) does not answer this question (at least not satisfactorily), nor does it give satisfactory justifications for the use of Nash equilibrium (NE)[3] in game theoretic models (Nash, 1951). The question of equilibrium selection is another problem area for TGT. Evolutionary game theory (EGT) attempts to come to the rescue of TGT by offering possible answers to the following questions:

(1) Does evolution wipe out irrational behaviour, that is, do individuals who play dominated strategies get eliminated from the population in the long run?
(2) Can the process of iterative elimination of (weakly) dominated strategies be justified by appealing to evolutionary processes?
(3) Can the use of Nash equilibria (Nash, 1951) be justified by appealing to an evolutionary process?
(4) Does evolution force coordination of the individual's rational actions?
(5) Can evolution lead to more refined notions such as subgame perfect equilibria?
(6) Does evolution resolve the problem of equilibrium selection?
(7) How does the evolution of cooperation emerge in repeated games?
(8) In static games, do evolutionary pressures lead to efficient equilibria?

Efforts at giving theoretically sound answers to questions (1) - (2), (3) - (5), and (6) - (7) are what Van Damme (1994) classifies under the headings "Evolution of rationality", "Evolution towards equilibrium" and "Evolution of norms" respectively. In this note, we review some of the papers in the literature that address these questions. The article is intended for those with background in economics and familiar with traditional game theory but not necessarily familiar with evolutionary game theory. We will therefore use concepts from traditional game theory without first having to explain them. We attempt to highlight the areas where the two branches of game theory converge, and more importantly, their areas of divergence. The focus is on the contributions of evolutionary game theory to the body of knowledge at the disposal of the game theorist. We do not attempt a highly

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[2]In game theoretic settings (and economic theory in general), a "decision-maker" is rational if he makes decisions consistently in pursuit of his objectives. It is assumed that each player's objective is to maximise the expected value of his own payoff, which is measured in some utility scales (Myerson, 1991).

[3]Recall that the basic idea of NE is, if a population as a whole is playing a NE, then unilateral deviations cannot help a player.

technical and detailed survey here.[4] Our main objective is to reach out to the numerous economists and indeed, other social scientists, who are not as yet initiated in the basic theory of evolutionary games.

The rest of the paper is organised as follows. In the next section, we discuss certain basic evolutionary game theory concepts. We also discuss the attempts being made by economists to extend and modify these basic concepts so as to make it applicable to the analysis of social interactions, in which individuals are conscious and try to use available information to their advantage. We outline the main problems of traditional game theory and review how the evolutionary game theoretic literature attempts to deal with them in section 3. In section 4, we discuss the possibility of applying evolutionary game theory in the analysis of economic problems and issues. Finally, section 5 concludes the paper.

2. Basic Evolutionary Game Theory

Evolutionary game theory came into existence with the work of Maynard Smith and Price (1973) which sought to explain why limited war behaviour is so prevalent among species which possess lethal weapons. Prior to this pioneering work, it was mostly believed that such limited war behaviour was due to group selection. Maynard Smith and Price (1973), using game theory showed that this limited war behaviour could also be due to individual selection. Since then, evolutionary game theory has been applied to several questions in evolution and biology.

In general, an evolutionary process consists of two basic elements: a *mutation mechanism*, which provides variety and a *selection mechanism*, which favours some varieties over others (Weibull, 1995). In evolutionary games, the criterion of *evolutionary stability* highlights the role of mutations, while that of *replicator dynamics* highlights the role of selection. All these concepts are borrowed from biological game theory (Maynard Smith and Price, 1973; Maynard Smith, 1974, 1982; Taylor and Jonker, 1978).

An *evolutionarily stable strategy (ESS)*[5] is robust to evolutionary selection pressures in an exact sense. Following the discussion in Weibull (1995), suppose individuals are repeatedly drawn at random from a large population to play a symmetric two-person game. Assume further that initially all individuals are genetically or otherwise "programmed" to play a certain pure or mixed strategy in this game. Now, let in a small population share of individuals who are likewise "programmed" to play some other pure or mixed strategy. Then the "incumbent" strategy is said to be *evolutionarily stable* if, for each such "mutant" strategy, there exist a positive *invasion barrier*[6] such that if the population share of

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[4]Interested readers are referred to Van Damme (1994), Banerjee and Weibull (1992), Hammerstein and Selten (1993), Binmore and Samuelson (1993) and Mailath (1992), for more detailed reviews of evolutionary game theory.

[5]See Hines (1987), and Hofbauer and Sigmund (1988) for good reviews of ESS.

[6]A strategy $x \in \Delta$ has an invasion barrier if there exists some $\bar{e}_y \in (0,1)$ such that inequality (2.1) holds for all strategies $y \neq x$ and every $e \in \left(0, \bar{e}_y\right)$.

individuals playing the mutant strategy falls below this barrier, then the incumbent strategy earns a higher payoff than the mutant strategy. Formally,

Definition 2.1 *A strategy* $x \in \Delta$ *is an **evolutionarily stable strategy (ESS)** if for every strategy* $y \neq x$ *there exists some* $\overline{e}_y \in (0,1)$ *such that*

$$(2.1) \qquad u\left(x, e_y + (1 - e)x\right) > u\left(y, e_y + (1 - e)x\right)$$

holds for all $e \in (0, \overline{e}_y)$.

In the above definition $x \in \Delta$ denotes incumbent strategy and $y \in \Delta$ denotes mutant strategy; $\varepsilon \in (0,1)$ is the share of mutants in the post-entry population; u(x,y) is the payoff to strategy x when played against strategy y; and Δ denotes the simplex of mixed strategies.

Inequality (2.1) states what biological intuition suggests: evolutionary forces select against the mutant strategy if and only if its post-entry payoff (i.e., fitness) is lower than that of the incumbent strategy. Now, let Δ^{ESS} be the set of evolutionarily stable strategies in the game under study, and Δ^{NE}, the set of symmetric Nash equilibrium strategies. It can easily be shown that (Weibull, 1995):

$$(2.2) \qquad \Delta^{ESS} \subset \Delta^{NE}$$

The criterion of ESS, however, requires more than just the condition (2.2). For if x is evolutionary stable, and y is an alternative best reply to x, then x has to be a better reply to y than y itself. Hence, a strategy $x \in \Delta$ is evolutionarily stable if and only if it meets the "first" and "second" order best reply conditions given in (2.3) and (2.4) below.

$$(2.3) \qquad u(y,x) \leq u(x,x) \quad \forall y$$

$$(2.4) \qquad u(y,x) = u(x,x) \Rightarrow u(y,y) < u(x,y) \quad \forall y \neq x.$$

Inequality (2.3) and (2.4) characterise evolutionary stability as originally defined (Maynard Smith and Price, 1973; Maynard Smith, 1974, 1982). Inequality (2.3) shows that (x,x) is a Nash equilibrium if x is an ESS, and because of (2.4) the set of ESS is a subset of the set of symmetric Nash equilibria. In fact, every ESS induces a proper and (hence, perfect) equilibrium (Van Damme, 1987, Theorem 9.3.4).

A number of points should be noted about the criterion of evolutionary stability. Firstly, the issue of rationality does not arise in this set up. Secondly, this criterion refers implicitly to a close connection between the payoffs in the game and the stability of an incumbent strategy in a population. The payoffs are supposed to directly reflect biological "fitness", usually taken to mean the expected number of surviving offsprings. Thirdly, just as with Nash equilibrium, the evolutionarily stable property does not explain how a population arrives at such a strategy. This is a very important issue and we discuss this a little further in

the next few paragraphs. Instead, it asks whether once reached, a strategy is robust to evolutionary pressures. In this regard, one can say that this criterion does not add much to the concept of Nash equilibrium. Fourthly, despite its biological stance, evolutionary stability also provides a relevant robustness criterion for human behaviours in a wide range of situations, including of course, many interactions in the realm of economics (Weibull, 1995). Fifthly, this criterion is somewhat too stringent, hence, the formulation of weaker evolutionary stability criteria such as *neutral stability* (Maynard Smith, 1982), which requires that no mutant earns more than the incumbent, as against, all mutants must earn less than the incumbent in ESS, and *robustness against equilibrium entrants* (Swinkels, 1992), which requires that no mutant earns the maximal payoff possible.[7]

An evolutionary stable strategy (ESS) (Maynard Smith and Price, 1973) is said to have dynamic stability if, when the values of the evolutionary parameters are perturbed so that the community strategy is displaced from the ESS strategy, the force of natural selection will restore the ESS. Most of the classical literature on evolutionary games does not address the issue of dynamic stability of an ESS under the influence of natural
selection. This may be largely due to the inherent complexity of the problem of evolution through natural selection. The phenomenon that an ESS may not be stable under the process of dynamic selection was first noted by Eshel and Motro (1981) who put forth the concept of continuously stable strategy (CSS). This concept has been used by several authors in addressing the issue of dynamic stability (see for example Eshel et. al., 1997; Eshel, 1996; Taylor, 1989; Lessard, 1990; Nowak, 1990; Christiansen, 1991; Metz et. al., 1996; Geritz et. al., 1998; Kisdi, 1999 and references therein).

Initially, it was generally believed that a strategy that is a CSS may be predicted to be the outcome of evolution through dynamic selection. However, some recent articles using a concept of evolutionary stability called evolutionary stable neighborhood invader strategy (ESNIS) discovered that there are CSSs that may not be the outcome of evolution through dynamic selection (Apaloo, 1997a, 1997b; Kisdi and Meszena, 1993, 1995; Ludwig and Levin, 1991; Lessard, 1990; Geritz et. al., 1998; Kisdi, 1999).

Geritz et. al. (1998) using the four evolutionary stability concepts (ESS, convergence stability, neighborhood invader strategy (NIS), and protected dimorphisms) provide a general framework for modelling adaptive trait dynamics. These four concepts are largely known to be independent of each other but we note that a strategy which is a NIS is also convergence stable (Apaloo, 1997a). However a strategy which is convergence stable need not be a NIS. It has been shown that a strategy which is a CSS may not be the outcome of evolution through dynamic selection even if no polymorphisms occur. This result is shown to be valid in single species evolution with a one-dimensional strategy chosen from a continuum. On the other hand, it has been shown that an ESNIS will be the outcome of evolution through dynamic selection if no polymorphisms occur (Apaloo, 1997a, and references therein).

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[7]See Weibull (1994) and Swinkels (1992) for more on neutral stability and robustness against equilibrium entrants.

Geritz et. al (1998) and Kisdi (1999) have provided some detailed account of trait dynamics in the neighborhood of an evolutionary singular point that is convergence stable but not an ESS in a monomorphic population. They show that such singular points lead to the occurrence of polymorphisms. They also extend the analysis to polymorphic populations. But even in the simple monomorphic population evolution, two traits in the opposite sides of an ESNIS may coexist. Appearance of a third trait results in an ecological dynamics involving three strategies whose outcome may be illusive. Note that these three strategies in the neighborhood of an ESNIS (thus a CSS) may have the kind of pairwise relationships given in the example on page 652 in Strobeck (1973) and therefore stable coexistence of the three strategies may be possible. It is not hard to see that predicting the outcome of trait dynamics is not an easy task. We note here that the literarature on evolutionary game theory in the biological context is rather large. We refer the reader to two good review articles by Lessard (1991) and Eshel (1996)

Note that so far we have been dealing with monomorphic populations. Now assume a polymorphic population and let $n_X(t)$ be the number of x-individuals at time t. Then, one period later, the number of x_1-individuals in discrete time is given by

$$(2.5) \qquad n_{x_1}(t+1) = n_{x_1}(t)\left(1 + \sum_x p_x u(x_1, x)\right)$$

where $p_X = p_X(t)$ denotes the probability of meeting x-types or the fraction of such a type in the population. The continuous time equivalent of (2.5) can be written as (Van Damme, 1994)

$$(2.6) \qquad \dot{n}_{x_1} = n_{x_1} u(x_1, p)$$

where $u(x, p) = \sum_x p_x u(x, x)$ denotes the expected fitness of the x-types in the population characterised by p. Let $n = \sum_x n_x$ and differentiate the identity $p_X n = n_X$ to obtain the following dynamics for population proportions

$$(2.7) \qquad \dot{p}_x = p_x \left(u(x, p) - u(p, p)\right) \quad \forall x$$

where $u(p, p) = \sum_x p_x u(x, p)$ denotes the average fitness of the population. Equation (2.7) is the famous *replicator equation* (Taylor and Jonker, 1978). This equation says that strategies grow in the population if they do better than average, else they get slowly wiped out with time. It can be seen that Nash equilibrium is a stationary point of the dynamical system. Conversely, each stable stationary point is a Nash equilibrium, and as shown by Bomze (1986), an asymptotically stable[8] fixed point is a perfect equilibrium. Indeed, if

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[8]A strategy x $\in \Delta$ is asymptotically stable if (1) x is Lyapunov stable, that is, if small perturbations do not disturb the stationarity of x; and (2) There is a neighbourhood W of x such that if the initial state is in W, then the state x(t) converges to x as t $\rightarrow \infty$.

inheritance of mixed strategies are allowed then asymptotically stable fixed points of the replicator dynamics corresponds exactly to ESS (Bomze and Van Damme, 1992; Hines 1980; Zeeman, 1981). However, if only pure strategies are inherited, being an ESS is sufficient, but not necessary for asymptotic stability (Taylor and Jonker, 1978).

Clearly, the replicator dynamics as given by equation (2.7) may be inadequate for modeling social interactions. For one thing, individuals possess some consciousness and try to use information to their advantage, and for another, cultural evolution through learning, imitation and experimentation may be governed by a different law of motion (Boyd and Richerson, 1985; Selten, 1991). Also, utility and profits in economics are not the same as reproductive fitness in biology, and the assumption of uniform random matching that underlines the replicator equation may be appropriate only in a limited number of economic contexts. Due to these problems, there is general consensus among economists that considerable amounts of technical development of these ideas and concepts must precede their applications in economics. Gladly, we can say that efforts in this direction have already produced some useful results (See for instance, Basu, 1988, 1990; Bomze, 1986; Nachbar, 1990; Samuelson and Zhang, 1992; Fudenberg and Harris, 1992; Fudenberg and Kreps, 1989; Friedman, 1991).

The basic model deals only with two-person, symmetric, static interactions. Clearly, this is a significant limitation of the model, since an asymmetric context with different populations appears to be more appropriate in economic contexts. Selten (1980), showed that, in an asymmetric context, the conditions analogous to (2.3) and (2.4) can be justified only at a strict Nash equilibrium.[9] It is, however, known that many games do not admit such equilibria, hence, they fail to have ESS. As a way to bypass this problem, concepts with better existence properties have been developed: Maynard Smith (1982) introduced the notion of neutrally stable strategy (NSS), which even though a weaker notion, has better existence properties. Furthermore, *set-valued* concepts have also been introduced (Thomas, 1985) for the same reason.
(corrected)

Definition 2.2 *$X \subset \Delta$ is an* **Evolutionarily Stable (ES) set** *if it is nonempty and closed and each $x \in X$ has some neigbhorhood U such that $u(x,y) \geq u(y,y)$ for all $y \in U$, with strict inequality if $y \notin X$.*

Yet another problem of the basic model is that, (2.4) might be a too stringent requirement in economic contexts, since it requires stability against all mutants, including "stupid" ones. When mutants arise through conscious experimentation, stupid mutants are unlikely to be introduced in the population, we might therefore be satisfied if only stability against "sensible" mutants are guaranteed.

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[9]Strict Nash equilibrium requires that any unilateral deviation actually incurs a loss. In a non-strict Nash equilibrium, at least one player is indifferent between some of his pure strategies even under his Nash equilibrium beliefs. Such indifference can make the equilibrium highly "unstable".

Definition 2.3 *(Swinkels, 1992) **"Sensible"** mutants can be described as best responses to the "perturbed" population in which they are present in small numbers, and an **equilibrium evolutionarily stable strategy (EES)** is a strategy that is stable against such mutants.* [10]

3. The Relationship Between Traditional and Evolutionary Game Theory

The relationship between the two types of game theory can be likened to the tension between the "mathematical" and the "economic", one between the normative and the descriptive, as described by Luce and Raiffa (1957, p.62-3). Traditional game theory would then be the normative, showing how agents should act if they wish to achieve certain aims, and evolutionary game theory, the descriptive, showing how they actually behave. One can claim that the key to traditional game theory's hegemony is the concept of Nash equilibrium.[11] In fact, given the results so far obtained by evolutionary game theorists (Bicchieri, 1989; Basu, 1988, 1990; Bomze, 1986; Nachbar, 1990; Samuelson and Zhang, 1992; Cabrales and Sobel, 1992; Banerjee and Weibull, 1992; Van Damme, 1987; Kandori, Mailath and Rob, 1993, Fudenberg and Maskin, 1991), the same can be said about EGT, that is, the concept of Nash equilibrium is central to it. While TGT gives a normative statement of Nash equilibrium, EGT attempts to give a descriptive justification of the concept, this then is one of the main differences between the two game theories.

The ascendancy of NE is probably due to the fact that it is an embodiment of the idea that economic agents are rational, and that they act simultaneously to maximise their utility. In this way NE embodies the most important and fundamental idea of economics (Aumann, 1985). Ironically, these very ideas of rationality and utility maximisation that EGT has exploited in order to establish itself as a separate and legitimate discipline. TGT simply assumes rationality, without giving good justifications for such assumptions. EGT, on the other hand, attempts to justify rationality through evolution, adaptation, experimentation, imitation, and learning.

The assumption of rationality and "common knowledge of rationality" inherent in TGT is not enough to generate NE behaviour (Banerjee and Weibull, 1992). In games with a dynamic structure, the very notion of rationality becomes problematic and common knowledge of rationality may even lead to logical contradictions (Rosenthal, 1981; Binmore, 1987; Bicchieri, 1989; Basu, 1988, 1990). EGT approaches this problem by asking the question, do economic agents behave *"as if"* they meet the stringent rationality and coordination assumptions inherent in TGT and the NE concept? The process of *natural selection* is one of the processes used to justify the "as if" approach. The main question

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[10]Swinkels (1992), also gives the set valued analogue of this concept.

[11]The justification for this claim may be found by noting comments made by giants in the field. For instance, Kreps (1990, p.1), notes that nowadays one cannot find a field of economics, finance, accounting, marketing, or even political science, in which understanding the concept of Nash equilibrium is not nearly essential. Tirole (1988) states that Nash equilibrium is "the basic solution concept in game theory". Rasmusen (1989), writes "Nash equilibrium is so widely accepted that the reader can assume that if a model does not specify which equilibrium concept is being used then it is Nash".

here is, does evolutionary selection among more or less boundedly "rational" behaviours in strategic interaction situations lead to NE? If the answer to this question is in the affirmative, then one can claim that whether or not players are genuinely "rational" and "co-ordinate their beliefs", in the long run they will behave as if they met the underlying rationalistic assumptions (Weibull, 1994).

A second approach to the justification of NE is through the use of *evolutionary adaptation*. In the basic dynamic setting of evolutionary adaptations, one imagines pairwise and randomly matched interactions in a large population of individuals, each interaction taking the form of play of a symmetric and finite two-player game. This approach lends strong support to the NE concept in such low-dimensional 2x2 symmetric games. Unfortunately, this result does not extend to general kxk games, because the evolutionary selection process in higher-dimensional spaces do not converge. However, the good news is, just as in the 2x2 case, if evolutionary selection induces no movement in the composition of a population's aggregate behaviour, and that behaviour is dynamically robust with respect to small perturbations, then it is compatible with the stringent rationality and coordination hypotheses in the rationalistic justification of NE behaviour. Bomze (1986), summarises this results in the proposition given below.

Proposition 3.1 (Bomze, 1986): If the population state x is (Lyapunov) stable in the replicator dynamics (2.7), then (x,x) is a Nash equilibrium. If (x,x) is a Nash equilibrium, then x is stationary in the replicator dynamics.

Just as in the 2x2 case, if the population state converges from an initial state in which all strategies are used, then the limiting state has to be a Nash equilibrium:

Proposition 3.2 (Nachbar, 1990): If an interior dynamic path in the replicator dynamics (2.7) converges to some $x \in \Delta$, then (x,x) is a Nash equilibrium.

Proposition 3.2 implies that every strictly dominated strategy is wiped out from the population provided all strategies are represented in the initial population and that the induced dynamic path converges.

Samuelson and Zhang (1992) went ahead to show that even when aggregate behaviour does not converge, all strictly dominated strategies will nevertheless be wiped out of the population.

Proposition 3.3 (Samuelson and Zhang, 1992): If a pure strategy is not rationalizable[12], then its population share converges to zero along any interior path in the replicator dynamics (2.7).

It is worth noting that it is not all smooth sailing for the three propositions stated above. For instance, Dekel and Scotchmer (1992) show, by way of an example, that proposition 3.3

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[12]See Bernheim (1984) and Pearce (1984) for definition of rationalizable strategies.

does not generally carry over to the discrete counterpart of the continuous time replicator dynamics. However, Cabrales and Sobel (1992) pointed out that the reason why the example of Dekel and Scotchmer works the way it does is not temporal discreteness per se, but the special form it is given. They went ahead to show that if the time discretization is made sufficiently "fine", such that only a small batch of individuals change strategy each time, then proposition 3.3 still stands, and all strictly dominated strategies are wiped out along all interior solution paths.

One class of dynamics that has been widely used in the literature is *monotone dynamics*, its popularity stems from the fact that it is a class of evolutionary dynamics that is wide enough in scope to handle plausible social interaction processes. It has the property that if some pure strategy earns more than another, then the first sub-population grows at a higher rate than the second. The replicator dynamics belongs to this class, but the difference now is that one may interpret monotone dynamics in terms of infinitely lived, boundedly rational individuals who consciously choose their strategy, or rather, revise their choice of strategy with time.

It can easily be shown that, in the special case of 2x2 games, the qualitative results of the replicator dynamics are shared by all monotone dynamics (Weibull, 1992). This implies that aggregate behaviour always converges to NE in these low-dimensional settings. Furthermore, it is possible to show that, even for arbitrary symmetric kxk games, many properties of the replicator dynamics are valid for any monotone dynamics (see Barnejee and Weibull, 1992).

We now summarize the discussion so far and give a hint on the direction it will take from now on. The concept of ESS was discussed in the first part of section 2, followed by a statement of the replicator dynamics equation. Note that in the case of the ESS, there is no dynamics, hence, the concept is a static one. With the replicator dynamics, however, there is explicit dynamics with only one-shot perturbation of stationary points, but explicit mutations are excluded. In what follows, we discuss noisy evolution, first introduced by Foster and Young (1990). Here, both explicit dynamics and mutations are included. Noisy evolution is synonymous with stochastic EGT, while ESS and the replicator dynamics are deterministic.

Foster and Young (1990) introduced the discussion on stochastic EGT by adding a stochastic term to the continuous replicator dynamics. Their motivation stemmed from the observation that the ESS criterion does not capture the notion of long-run stability in a stochastic dynamical system. They saw a limitation in the assumption of the ESS criterion, where each perturbation is treated as if it were an isolated event: in reality a system is continually being subjected to perturbations that arise through mutations, such that over the long run, it is likely that some succession of perturbations will accumulate and kick the system out of any immediate locus of an ESS. Their major point is to show that the introduction of stochastic effects may qualitatively change the asymptotic behaviour of an evolutionary system.

Traditional game theory has not been successful in explaining how players know which NE is to be played if a game has multiple and equally plausible NE. The question of equilibrium selection is so troublesome that it deserved mention by Kreps (1990) as one of the problems of TGT. In our view stochastic EGT appears to offer the greatest opportunity of solving this problem. Foster and Young (1990) were the first to argue that in games with multiple equilibria and therefore multiple ESS, some equilibria are more likely to emerge than others in the presence of continual small stochastic shocks. This point was apparently taken seriously by Kandori, Mailath, and Rob (1993). In their paper they introduced a discrete framework to address the issue of equilibrium selection, and developed a technique for determining the most likely or long-run equilibrium. They went ahead to implement their technique on coordination games, where they show that for this kind of games, the long-run equilibrium coincides with the *risk dominant equilibrium* of Harsanyi and Selten (1988). A follow up to the work of Kandori, Mailath and Rob (1993) is Ellison (1993). This paper among other things, concentrated on the rate of convergence of dynamical systems. Ellison rightly pointed out that in economic applications the rate of convergence of play to its long run limit, may be the main measure of the relevance of the evolutionary forces causing the convergence. In this regard, they came to two conclusions. First, the nature (local or global) of interactions in a population is a crucial determinant of play. Second, when interactions are local, the evolutionary arguments of Kandori, Mailath and Rob (1993) may be reasonably applied to large populations.

Another interesting contribution in this direction is Young (1993a): Consider an n-person game that is played repeatedly, but by different agents. Young applies a technique which defines a stochastic process that, for a large class of such games, converges almost surely to a pure strategy NE or what is called "convention" in the paper. If the players in the game sometimes make mistakes or experiment, then the society occasionally switches from one convention to another. As the chance of making mistakes goes to zero, only some conventions, known as *stochastically stable equilibria*, have a positive probability in the limit.

4. Some Possible Application Areas

With its emphasis on "as if" rationality, imitation, learning and evolutionary trial-and-error processes involving players that are boundedly rational, we can foresee great potential in the application of evolutionary game theory in the analysis of problems in economics and related subject areas, in general. In particular, we see EGT to be of relevance to the study of both economic under-development and natural resource management. How this prediction will materialise and the form it will take, we do not know at the moment, but the potential is clearly there. For instance, it is not too difficult to see that evolutionary game theory can be used profitably in the analysis of the behaviour of agents in a market for transferable fishing quotas. It is possible to envisage a market equilibrium with one big survivor, constituting the fittest among the agents and a number of small survivors. The existence of certain locally based advantages such as good proximity to a village fish market or access to some valuable but small stocks of certain species of fish, could be possible reasons for the survival of the latter group of fishers in such a scenario.

Another possible application: consider a group of commercial fishing companies, all operating in the same habitat, say the Barents Sea. Assume that each of them is boundedly rational, that is, they are myopic and have short memories; naive and perform very little optimization exercise; have no knowledge of the stock sizes or stock dynamics of the fishes they exploit; and have little ability to observe rival actions and their consequences. Assume further that to decide what fishing effort to exert in any fishing period, all that a typical agent does is to note the marginal profit he earned in the preceding period, if this turns out to be positive, then he adjusts his effort level positively, if, on the other hand, his marginal profit turns out to be negative, he will decrease his effort level, and keeps his preceding period fishing effort unchanged if this marginal profit is zero. It is possible to formalise this problem to show that such a group of fishers may evolve, pushed by evolutionary forces, to (possibly) a unique Nash equilibrium in the long run.

5. Concluding Remarks

Two features of evolutionary game theory as opposed to traditional game theory emerges from this survey. First, players are not assumed to be so "rational" or "knowledgeable" as to be in a position to anticipate the other player's choices correctly. Second (and instead), an explicit dynamic process is specified which describes how players adjust their choices over time as they learn from experience about other players choices and the structure of the game itself. Thus the EGT approach tries to explain how an equilibrium is reached based on trial-and-error learning instead of introspective-type arguments as advanced in traditional game theory.

One can divide the development path of EGT into two phases. Phase one would then be concerned with the theoretical and technical development of the discipline, while the second phase will be concerned with both the application in economic contexts, and the further theoretical and technical development of the field. The discussion in sections 2 and 3 show that phase one has already reached an advanced stage. And indications are that phase two has already taken off. A solid indication of this is the work of Young (1993b), where individuals from two different populations (landlords and tenants in their model) of bargainers are randomly matched to play the Nash bargaining game. We believe it is time to give momentum to applications of evolutionary game theory in the analysis of economic problems, in particular, environmental and natural resource management problems.

Reference list

Apaloo, J. (1997a). Revisiting strategic models of evolution: the concept of neighborhood invader strategies. *Theoretical Population Biology* **52** (1) 71–77.

Apaloo, J. (1997b). Ecological species coevolution. *Journal of Biological Systems* **5** (1) 17–34.

Aumann, R (1985): What is game theory trying to accomplish? In Arrow and Honkapohja (eds.) *Frontiers in Economics*, Basil Blackwell.

Banerjee, A. and J. W. Weibull (1992): *Evolution and rationality: Some recent game-theoretic results.* Proceedings from the Tenth World Congress of the International Economic Association, Blackwell (forthcoming).

Basu, K. (1988): *Strategic irrationality in extensive games,* Mathematical Social Sciences 15 247-260.

Basu, K. (1990): On the non-existence of a rationality definition for extensive games. *International Journal of Game Theory*, 19, 33-44.

Bernheim, D., (1984): Rationalizable strategic behavior. *Econometrica,* 52 1007-1028.

Bicchieri, C. (1989): Self-refuting theories of strategic interaction: a paradox of common knowledge. *Erkenntnis*, 30 69-85.

Binmore, K (1992): *Fun and games: A text on game theory,* Chancillor Press, London.

Binmore, K. (1987): Modeling rational players, Part 1, *Economics and Philosophy* 3(2) 179-214.

Bomze, I. A. (1986): Non-cooperative two-person games in biology: a classification, *International Journal of Game Theory,* 15, 31-57.

Bomze, I. M. and E. Van Damme (1992): A dynamical characterization of evolutionary stable states. *Annals of Operations Research,* 27, 229-244.

Boyd, R. and P. Richerson (1985): *Culture and the evolutionary process.* Univ. of Chicago Press, Chicago.

Cabrales, A. and J. Sobel (1992): On the limit points of discrete selection dynamics. *Journal of Economic Theory,* 57, 407-419.

Christiansen, F. B. (1991). On conditions for evolutionary stability for a continuously varying character. *The American Naturalist* **138** (1) 37–50.

Dekel, E. and S. Scotchmer (1992): On the evolution of optimizing behavior. *Journal of Economic Theory,* 57, 392-406.

Ellison, G. (1993): Learning local interaction, and coordination. *Econometrica*, 61, 1047-1072.

Eshel, I. (1996). On the changing concept of evolutionary population stability as a reflection of a changing point of view in the quantitative theory of evolution *Journal of Mathematical biology* **34** 485–510.

Eshel, I., and Motro, U. (1981). Kin selection and strong evolutionary stability of mutual help *Theoretical Population Biology* **19** 420–433.

Eshel, I., Motro, U., & Sansone, E. (1997). Continuous stability and evolutionary convergence J. theor. Biol. **185**, 333-343.

Foster, D. and P. Young (1990): Stochastic evolutionary game dynamics. *Theoretical Population Biology* 38, 219-232.

Friedman, D., (1991): Evolutionary games in economics. *Econometrica*, 59 637-666.

Fudenberg, D., and C. Harris (1992): Evolutionary dynamics with aggregate stock shocks. *Journal of Economic Theory,* 57, 420-441.

Fudenberg, D., and D. Kreps (1989): *A theory of learning, experimentation and equilibrium in games*, Mimeo, MIT and Stanford.

Fudenberg, D., and E. Maskin (1991): Evolution and cooperation in noisy repeated games. *American Economic Review.* 80, 274-279.

Geritz, S. A., Kisdi, E., Meszϑna, G., and Metz, J. A. J. (1998). Evolutionarily singular strategies and the adaptive growth and branching of the evolutionary tree *Evolutionary Ecology* Eshel, I., Motro, U., and Sansone, E. Continuous stability and evolutionary convergance **12** 35–57.

Hammerstein, P. and R. Selten (1993): Game theory and evolutionary biology. Mimeo, Max-Planck-Institute, Seewiesen and Dept. of Economics, Bonn Univ.

Harsanyi, J. C. and R. Selten (1988): *A general theory of equilibrium selection in games*. MIT Press, Cambridge MA.

Hines, W. G. S. (1980): Three characterizations of population strategy stability. *Journal of Applied Probability*,17, 333-340.

Hines, W. G. S. (1987): Evolutionary Stability Strategies: A Review of Basic Theory. *Theoretical Population Biology,* 31, 195-272.

Hofbauer, J. and K. Sigmund (1988): *The theory of evolution and dynamical systems*. Cambridge Univ. Press, Cambridge.

Kandori, M., G. Mailath and R. Rob (1993): Learning, mutations, and long run equilibria in games. *Econometrica,* 61, 29-56.

Kisdi, E. (1999). Evolutionary branching under asymmetric competition *J. Theor. Biol.* **197** 149–162.

Kisdi, E., and Meszϑna, G. (1993). Density dependent life history evolution in fluctuating environments. In *Adaptation in a stochastic environment – Lecture Notes in Biomathematics* J. Yoshimura and C. Clark, Eds. Springer Vatag pp. 26–62.

Kisdi, E., and Meszϑna, G. 1995. Life histories with lottery competition in a stochastic environment: Esss which do not prevail *Theoretical Population Biology* **47** 191–211.

Kreps, D. (1990): *Game theory and economic modeling*, Oxford Univ. Press, Oxford.

Lessard, S. (1990). Evolutionary stability: one concept, several meanings *Theoretical Population Biology* **37** 159–170.

Ludwig, D., and Levin S. A. (1991). Evolutionary stability of plant communities and the maintenance of multiple dispersal types *Theoretical Population Biology* **40** 285–307.

Luce, R. D. and Raiffa, H. (1957): *Games and decisions*, New York, Wiley.

Mailath, G. (1992): Introduction: Symposium on evolutionary game theory. *Journal of Economic Theory*, 57, 259-277.

Maynard Smith, J. and G. R. Price (1973): The logic of animal conflict. *Nature,* 246, 15-18.

Maynard Smith, J. (1974): The theory of games and the evolution of animal conflicts. *Journal of Theoretical Biology,* 47, 209-221.

Maynard Smith, J. (1982): *Evolution and the theory of games*, Cambridge Univ. Press, Cambridge.

Metz, J. A. J., Geritz, S. A. H., Meszϑna, G., Jacobs, F., and van Heerwaarden, J. S. January (1995). Adaptive dynamics, a geometrical study of the consequences of nearly faithful reproduction. In *Proceedings of the Colloquim, Amsterdam* S. J. van Strien and S. M. V. Lunel, Eds. North-Holland pp. 183–231.

Myerson, R.B. (1991): *Game theory*. Cambridge MA: Harvard Univ. Press.

Nachbar, J. (1990): Evolutionary selection dynamics in games. convergence and limit properties. *International Journal of Game Theory,* 19, 59-89.

Nash, J. R. (1951): Non-cooperative games, *Annals of Mathematics,* 54, 286-295.

Nowak, M. (1990). An evolutionarily stable strategy may be incaccessible *J. Theor. Biol.* **142** 237–241.

Pearce, D., (1984):Rationalizable strategic behavior and the problem of perfection, *Econometrica,* 52, 1025-1050.

Rasmusen, E. (1989): *Games and Information*, Oxford, Basil Blackwell.

Rosenthal, R. (1981): Games of perfect information, predatory pricing and the chain-store paradox. *Journal of Economic Theory,* 25, 92-100.

Samuelson, L. and J. Zhang (1992): Evolutionary stability in asymmetric games. *Journal of Economic Theory*, 57, 363-391.

Selten, R. (1980): A note on evolutionary stable strategies in asymmetric animal conflicts. *Journal of Theoretical Biology*, 84, 93-101.

Selten, R.(1991): Evolution, learning and economic behaviour. *Games and Economic Behavior,* 3, 3-24.

Strobeck, C. (1973). N species competition *Ecology* **54** (3) 650–654.

Swinkels, J. M. (1992): Evolutionary stability with equilibrium entrants. *Journal of Economic Theory*, 57, 306-332.

Taylor, P. D. (1989). Evolutionary stability in one-parameter models under weak selection *Theoretical Population Biology* **36** 125–143.

Taylor, P.D. and L.B. Jonker (1978): Evolutionarily stable strategies and game dynamics. *Mathematical Bioscience,* 40, 145-156.

Tirole, J. (1988): *The theory of industrial organisation*, Cambridge Mass. M.I.T. Press.

Thomas, B. 1(985): On evolutionarily stable sets. *Journal of Mathematical Biology,* 22, 105-115.

Weibull, J. W. (1992): An introduction to evolutionary game theory. Dept of Economics, Stockholm University, mimeo.

Weibull, J. W. (1994): The "As if" approach to game theory: 3 positive results and 4 obstacles, *European Economic Review,* 38, 868-881.

Weibull, J. W. (1994): *Evolutionary Game Theory*. Cambridge (USA): MIT Press (1995).

Van Damme, E. (1987): *Stability and perfection of Nash equilibria*, Springer Verlag, Berlin. Second Ed.

Van Damme, E. (1994): Evolutionary game theory, *European Economic Review,* 38, 847-858.

Young, P. (1993b): The evolution of conventions. *Econometrica,* 61, 57-84.

Young, P. (1993b): An evolutionary model of bargaining. *Journal of Economic Theory,* 59, 145-168.

Zeeman, E. C. (1981): Dynamics of the evolution of animal conflict. *Journal of Theoretical Biology*, 89, 249-270.

Summary

This note reviews the game theoretic literature with the aim of highlighting the similarities and dissimilarities between what we term traditional game theory and evolutionary game theory. The focus is on the contributions of evolutionary game theory to the body of knowledge at the disposal of the game theorist. The note is intended for people with interests in economics and who are familiar with traditional game theory but not necessarily familiar with evolutionary game theory. The main objective is to reach out to the numerous economists and indeed, other social scientists, who are not as yet initiated in the basic theory of of evolutionary games. A major conclusion of this note is that, applications of evolutionary game theory in the analysis of economic problems, especially, in the areas of natural, environmental and development economics are long overdue.

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