Non-Cooperation in Fish Exploitation

The Case of Irreversible Capital Investment in the Arcto-Norwegian Cod Fishery

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WP 1994: 2

Working Paper
Chr. Michelsen Institute
Development Studies and Human Rights
Bergen Norway

ISSN 0804-3639

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Summary:

A two-stage, two-player non-cooperative game model is developed under an irreversible capital investment assumption. The main aim is to predict the number of vessels that each player in such a game will find in his best interest to employ in the exploitation of the Arcto-Norwegian cod stock, given a non-cooperative environment and the fact that all players are jointly constrained by the population dynamics of the resource. The predictions obtained are then compared with (i) the sole owner's optimal capacity investments for the two players; (ii) the results in Sumaila (1994), where perfect malleability of capacity is assumed implicitly; and (iii) available data on the Arcto-Norwegian cod fishery.

Sammendrag:

Et to-steg to-aktør ikke-kooperativt spill er utviklet under en antagelse om irreversibel kapital investering. Hovedmålet er å finne antall fiskebåter som hver aktør finner i sin egen interesse å sette inn for å høste av den norsk arktiske torsken, gitt begrensninger i ressurstilgangen og at aktørene ikke samarbeider. Resultatene som framkommer er sammenliknet med (1) resultatet når bare en av aktørene har rettigheten til ressursen, (2) resultatene i Sumaila (1994) der man har en reversibel kapital antagelse, og (3) tilgjengelig data på de norsk arktiske torskefiskeriene.

Indexing terms:

Game theory
Fishery
Fishery resources
Coastal vessels
Cod
Trawlers
Norway

Stikkord:

Spillteori
Fiske
Fiskeressurser
Kystbåter
Torsk
Trål
Norge

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1.0 Introduction

This paper considers non-cooperative use of a common property fish stock, namely the Arcto-Norwegian cod. Attention is focused on a restricted access fishery where only two-agents participate in the exploitation of the resource, the aim being to predict the number of vessels that each agent in such a situation will find in his best interest to employ. An important although self-evident aspect of the game is that both agents are jointly constrained by the population dynamics of the resource. The key assumption of the paper is that players undertake investment in capital that is irreversible. This assumption is quite realistic because capital embodied in fishing vessels is often non-malleable: Non-malleability is used here to refer to the existence of constraints upon the disinvestment of capital assets utilized in the exploitation of the resource (Clark et al. 1979). This implies that once a fishing firm or authority invests in a fleet of vessels it either has to keep it until the fleet is depreciated, or else the vessels can only be disposed off at considerable economic loss.

A number of papers have appeared in the fishery economics literature that focus, among other things, on the irreversibility of capital employed in the exploitation of fishery resources. Examples include Clark et al. (1979), Clark & Kirkwood (1979), Dudley & Waugh (1980), Charles (1983a, 1983b), and Charles & Munro (1985). We are, however, not aware of any prior work that models, computes numerically and analyses the exploitation of fishery resources as done in this paper. Among the examples cited above, only Dudley & Waugh (1980) consider investment decision in a fishery with more than a single agent participating. But even in this case, only qualitative statements of the likely effects of this are made. The study of Clark & Kirkwood (1979) is close to the work planned herein, at least in terms of the kind of questions they address. The authors presented a bioeconomic model that predicts the number of vessels of each of the two types entering the prawn fishery of the Gulf of Carpentaria under free access. In addition, they estimated the economically optimal number of vessels of each type. The results they obtained are then compared with available data on the prawn fishery of the Gulf of Carpentaria.

These are issues we also address here albeit with a number of differences. First, there is a difference with respect to the number of agents in the two studies: While Clark & Kirkwood (1979) consider the social planner's and open access equilibrium fleet sizes, we compute equilibrium fleet sizes that will emerge in a non-cooperative environment involving two agents, and then, using these results, we derive the social planner's equilibrium fleet size and discuss the probable open access equilibrium fishing capacity. Thus, we add a new dimension to the discussion, namely, the two-agent analysis[1]. Second, there is a difference in the way we model the population dynamics of the fish stock: While their study prescribes and uses a single cohort to describe the fish stock, we accommodate a multicohort population structure.

The primary concern of this study is to develop the necessary framework to

(1) identify a Nash non-cooperative equilibrium solution for a bimatrix game involving the trawl and coastal fisheries operating on the Arcto-Norwegian cod;

[1] The motivation for undertaking a two-agent analysis is given in section 2 of this paper.

(2) identify the sole owner equilibrium solutions for the two fisheries, and determine which among these gives the optimal solution;

(3) compare the results in (1) and (2) above to (i) the results in Sumaila (1994), where perfect malleability of capital is assumed implicitly, and (ii) with available data on the Arcto-Norwegian cod. The former comparison would put us in a position to say something about the possible gains of establishing rental firms for fishing vessels and/or allowing mobility of vessels between different stocks;

(4) discuss the fishing capacities that are likely to emerge in an open access scenario; and

(5) investigate the effect of fixed cost, interest rates, initial stock size, and the terminal constraint, on the relative profitability of the players.

The next section gives a brief description of the Arcto-Norwegian cod fishery. Section 3 presents the model, a special feature of which is the explicit modeling of the biologically and economically important age groups of cod. This is followed by a brief mention of the algorithm for the computation of the equilibrium solutions: The detailed algorithm is relegated to an appendix. In section 5, the results of the study are stated. Finally, section 6 concludes the paper.

2.0 The Arcto-Norwegian cod fishery

The Arcto-Norwegian cod, *gadus morhua,* is a member of the Atlantic cod family, arguably among the world's most important fish species. It inhabits the continental shelf from shoreline to 600 m depth, or even deeper, usually 150-200 m. It is gregarious in behaviour, forming shoals or schools and undertaking spawning and feeding migrations. The diet of adult cod is variable and consists mainly of herring, capelin, haddock and codling. The Arcto-Norwegian cod spawns only along the Norwegian coast, mainly in Lofoten in April-March. Typically, it starts spawning at the age of 7-8 years; eggs are carried by the gulf stream, over the coast where they hatch, and into the Barents Sea, up towards Svalbard, where the young cod grow. It has a relatively long life span: it can live for well over 15 years. A majority of young cod die quite early, either because of a lack of adequate food, or because they are eaten up by other fishes. Young cod between the ages of 3-6 come to the Finnmark's coast every year. This is because mature capelin, which cod preys on, move to their spawning spots close to the Finnmark's coast. Cod follows and predates them, thus resulting in good spring cod in the period April to June.

The Arcto-Norwegian cod is a shared resource jointly managed by Norway and Russia. Norwegian fishers employ mainly coastal and trawl fishery vessels in the exploitation of the resource, while their Russian counterparts employ mainly trawlers. Table 2.1 gives the number of Norwegian trawl and coastal fishery vessels (of 13m longest length and over) that operated on the "cod fishes group[2]" for five different years. In addition to this comes the part of the fishing capacity employed to exploit

[2] The cod fishes group include, the Arcto-Norwegian cod, the Arcto-Norwegian haddock, whiting, Greenland halibut, saith, etc.

other species, say, the "herring fishes group" that are used to land the cod fishes as bycatch.

[Table 2.1 in here!]

Using Norwegian data[3], we calculated the number of coastal fishery vessels and trawlers used by Norwegian fishers in the exploitation of the cod fishes group in 1991 to be about 638 and 58, respectively. These landed about 130 and 270 thousand tonnes of cod, respectively.

To facilitate our analysis three simplifications (about this fishery) are made[4]. First, only Norwegian prices and costs are used in the analysis. Second, the vessel types employed in the exploitation of the resource are grouped into two broad categories, namely, the coastal and the trawl fisheries, and placed under the management of two separate and distinct management authorities, henceforth to be known as Coastal Fisheries Management (C), and Trawl Fisheries Management (T). Third, only the *most cost effective* vessels[5] in each of these categories are assumed to be employed in the exploitation of the resource. The assignment of two separate and distinct fleets to the two management authorities captures, to some extent, the division of the stock between Norway and Russia, but even in Norway a division is usually made between the coastal fleet and the trawlers, and the Norwegian quota is divided between these.

3.0 The model

The model presented here builds on that discussed in Sumaila (1994), to which the reader is referred for details. Here, a two-stage, two-player, dynamic, deterministic, non-cooperative game model is put together, the two players being T and C. By a game we mean a normal (strategic) form, simultaneous-move game in which both players make their investment decisions in ignorance of the decision of the other. At stage one of the game, each player invests in fishing capacity *ex ante*, having in mind that such investment is irreversible. Then in stage two the players employ their chosen capacity investment to exploit the shared resource for the next 15 years, subject to the stock dynamics and nonnegativity constraints.

Both T and C are assumed to be rational and act here to maximize their discounted profit (payoff) function $\Pi_i : K_T \times K_C \rightarrow \Re$, where K_T and K_C are the pure strategy sets of player i = T, C, that is, the set of fishing capacity (number of vessels or fleet size) that a player can choose from. Player i's payoff at an outcome (k_T, k_C) is then given by $\Pi_i(k_T, k_C)$. A major aim of this modeling exercise is to find the strategy pair (k_T^*, k_C^*) such that no player will find it in his interest to change strategy given that his opponent keeps to his. In other words, we are interested in finding *Nash non-cooperative equilibrium* in a two-player fishery game, where k_T^* is a best reply to k_C^* and vice versa. This is equivalent to stipulating that the inequalities

[3] Data in tables E21 - E51 in Lønnsomhetsundersøkelser (1979 - 1990) were used for the calculations.
[4] See Sumaila (1994) for the justifications for these simplifications.
[5] Cost effectiveness is defined here in terms of least cost per kilogram of fish landed.

$$\Pi_T(k_T^*,k_C^*) \geq \Pi_T(k_T,k_C^*)$$

(1) $$\Pi_C(k_T^*,k_C^*) \geq \Pi_C(k_T^*,k_C)$$

hold for all feasible k_T and k_C.

3.1 On existence of Nash equilibrium

Nash (1950, 1951) proved the existence of equilibrium points under certain assumptions on each player's strategy space and corresponding payoff function. Essentially, he dealt with matrix games. Rosen (1965) went further to show that when every joint strategy lie in a *convex, closed,* and *bounded* region in the product space and each player's payoff function Π_i, i = T, C is *concave* in his own strategy and *continuous* in all variables, then there is at least one Nash equilibrium of the game. This result is stated in theorem 1 below.

THEOREM 1 (Existence of Nash equilibrium, Rosen (1965)): *An equilibrium point exist for every concave n-person game.*

The game we formulate in this paper is a concave 2-person game, and hence satisfies the above theorem. We can therefore expect at least one Nash equilibrium to exist in our game.

3.2 On uniqueness of Nash equilibrium

Two steps are taken here to deal with the vexing problem of equilibrium selection.

Step 1: Only open loop strategies are allowed in the second stage of the game[6]. That is, each player commits, in advance, his fishing capacity to a fixed time function rather than a fixed control law (closed loop strategies). Note that unlike in the case of fixed control laws, where the choice of control depends on the past history of the game, fixed time functions are independent of the actions of the opponent so far in the game. In information theoretic sense open loop corresponds to the receipt of no information during play, while closed loop represents full information.

The main reason we stick to open loop strategies, even though it is not likely to lead us to closed form solutions, is that it would be practically impossible to compute the predictions of our model if closed loop strategies were allowed. This is because closed loop strategies normally entail complex and huge numbers of strategies in repeated games (see Binmore, 1982). Another reason is that the new "Folk Theorem for Dynamic Games" introduced by Gaitsgory and Nitzan (1994), gives us reason to believe that under certain monotonicity assumptions, the set of closed loop solutions that may emerge from our model may coincide with the open loop solutions we compute herein.

[6] In a sense one can argue that the game we formulate herein is not a "pure" open loop strategy game. This is because although the fishing capacities are chosen once and for all, the capacity utilization is chosen in each period depending on the stock size.

Step 2: The same shadow prices (that is, Lagrange multipliers) are imposed across both players for resource constraint violation. Flåm (1993) shows that in addition to this, if the marginal profit correspondence is *strictly monotone*, then there exists a unique Nash equilibrium for our game. Incidentally strict monotonicity of marginal profit correspondence is also a sufficient condition for convergence in our model.

In the presentation of the mathematical equations in the rest of the model, two other subscripts (a=0,...,A, and s=1,...,S) are used to denote age groups of fish, and time periods or stages, respectively[7]. Based on the life expectancy of cod, the last age group A, is set equal to 15. The finite time horizon of the game, S, is set equal to 15 due to computational limitations.

Catch

Let catch of age group *a* (in number of fish) by player *i* in fishing period *s*, $h_{i,a,s}$, be given by

(2) $$h_{i,a,s} = q_{i,a} n_{a,s} E_{i,s}$$

where the effort profile, $E_{i,s} = k_i e_{i,s}$, and k_i is the *ex ante* fixed capacity investment of player *i*; $e_{i,s} \in [0,1]$, is the capacity utilisation, that is, the fraction of k_i taken out for fishing in a given year; $n_{a,s}$ is the post catch number of fish of age *a* in fishing period *s* and $q_{i,a}$ is the player and age-dependent catchability coefficient, that is, the share of age group *a* cod being caught by one unit of effort.

Total catch by all players of age group *a* in period *s* can thus be written as

(3) $$h_{a,s} = \sum_i q_{i,a} n_{a,s} k_i e_{i,s}$$

Total catch in weight by all players over all age groups in period *s* is given by

(4) $$h_s = \sum_a h_{a,s} w_a$$

where w_a is the weight of fish of age group *a*.

Costs and Prices

The fishery is assumed to face perfectly elastic demand. Thus, the ex-vessel selling price of fish per kilogram, V, is assumed to be constant. The harvesting costs per vessel employed by player *i* in period *s*, $\psi_{i,s}$ consist of fixed costs (φ_i) and variable costs (ϑ_i) which are proportionate to $e_{i,s}$,

(5) $$\psi_{i,s} = \varphi_i + \frac{\vartheta_i}{(1+b)} e_{i,s}^{1+b}$$

---

[7] Recall that the subscript denoting player is i = T, C.

where b = 0.01. This formulation of the cost function ensures strict concavity of individual profit as a function of individual effort. This strictness is important for the sake of convergence.

Revenue

The revenue to player *i*, in period *s*, $r_{i,s}$, comes from the sale of his catch over all age groups in that period, that is,

$$r_{i,s} = V \sum_a w_a h_{i,a,s} \tag{6}$$

Profits

Player *i*'s profit in a given period *s* is then given by the equation

$$\pi_{i,s}(n_s, e_{i,s}) = r_{i,s} - k_i \psi_{i,s} \tag{7}$$

where $k = (k_i, k_{-i})$. Note that $\pi_{i,s}$ is a function of the actual fish abundance in a period, n_s, and own effort of a player in that period. We restrict our analysis to the case of perfectly non-malleable capital in which the depreciation rate is equal to zero and capital has a negligible scrap value. Even though this simplification is not quite realistic, the qualitative effect of this is expected to be insignificant. The profit function given by equation (7) is formulated to incorporate this restriction.

Objective

Given k_i, the second stage problem of player *i* is to find a sequence of capacity utilization, $e_{i,s}$ (s=1,2,...,S), to maximise his present value (PV) of profits (payoff), that is,

$$\Pi_i(k_T, k_C) = \max_{e_i} \sum_{s=1}^{S} \delta_i^s \pi_i(n_s, e_{i,s}) \tag{8}$$

subject to the stock dynamics (see below) and the obvious nonnegativity constraints, expressed mathematically as $e_{i,s} \geq 0$, for all *i* and *s;* $n_{a,s} \geq 0$, for all *a* and *s;* $n_{a,S+1} \geq 0$, for all *a*; and $n_{a,0} \geq 0$ given. Here, $\delta_i = (1 + r_i)^{-1}$ is the discount factor; $r_i > 0$ denotes the interest rate of player *i*; and $n_{a,0}$ is a vector representing the initial number of fish of each age group.

An important but self-evident component of this game is that players are jointly constrained by the population dynamics of the fish stock. Nature is introduced (as a player) in the game with the sole purpose of ensuring that the joint constraints are enforced. The decision variable of nature is thus the stock level - its objective being to ensure the feasibility of the stock dynamics. Formally, nature's objective is expressed as 0 if the stock dynamics is feasible, and $-\infty$ otherwise.

It is worth mentioning here that unless players enjoy bequest, they will typically drive the fishable age groups of the stock to the open access equilibruim level at the end of the game, if the terminal restriction is simply $n_{a,S+1} \geq 0, \ \forall a$. To counteract this tendency, one can exogeneously impose the more restrictive constraint, $n_{a,S+1} \geq \tilde{n}_a$, where $\tilde{n}_a$ is a certain minimum level of the stock of age group *a* that must be in the habitat at the end of period S+1. This is what is done here. Alternatively, this restriction can be imposed endogeneously by obliging the players to enter into a stationary regime maintaining constant catches and keeping escapment fixed from S onwards.

Stock dynamics
Let the stock dynamics of the biomass of fish in numbers $n_{a,s}$, (that is, the joint constraint mentioned above) be described by

$$
\begin{aligned}
& n_{0,s} \leq f(B_{s-1}), \\
& n_{a,s} + h_{a,s} \leq \xi_{a-1} n_{a-1,s-1}, \qquad \text{for} \quad 0 < a < A \\
& n_{A,s} + h_{A,s} \leq \xi_A n_{A,s-1} + \xi_{A-1} n_{A-1,s-1},
\end{aligned}
\tag{9}
$$

where $f\left(B_{s-1} = \dfrac{\alpha B_{s-1}}{1+\gamma B_{s-1}}\right)$ is the Beverton-Holt recruitment[8] function; $B_{s-1} = \sum_a p_a w_{sa} n_{a,s-1}$ represents the post-catch biomass in numbers; p_a is the proportion of mature fish of age *a*; $w_{s,a}$ is the weight at spawning of fish of age *a*; α[9] and γ are constant parameters chosen to give a maximum stock size of about 6 million tonnes - a number considered to be the approximate carrying capacity of the habitat[10]; ξ_a is the natural survival rate of fish of age *a*; and $h_{a,s}$ defined earlier, denotes the combined harvest of fish of age *a*, in fishing season *s* by all agents.

4.0 Numerical method

An algorithm is developed to resolve the second stage game problem, that is to find the answer to the following question: given the fixed capacity choice of the players in stage one of the game, what level of capacity utilization should they choose in each fishing period so as to maximize their respective economic benefits? For detailed discussions of the theoretical basis for the algorithm see in particular Flåm (1993), and also Sumaila (1993). In what follows, we briefly outline the conceptual algorithm and describe in concrete terms its practical implementation, the detailed problem-specific algorithm is relegated to an appendix.

[8] In this model, recruitment refers to the number of age zero fish that enter the habitat in each fishing period.

[9] $\alpha = f'(0)$, is the number of recruits per unit weight of biomass "at zero" or the polutation level.

[10] Researchers at the Institute of Marine Research, Bergen, estimate the maximum sustainable yield (MSY) stock level to be about 3 million tonnes: With an assumption that the MSY stock level is one half of the pristine stock level we get the figure of 6 million tonnes.

Suppose for illustrative purposes that all constraints (except nonnegativity ones) are incorporated into one concave restriction of the form $\Phi(n, e_i, e_{-i}) \geq 0$, where e_{-i} is the profile of capacity utilisation of *i*'s rival, e_i is the equivalent for player *i*, and *n* is the stock profile (note that $n \in R^{(A+1),S}$, and $e_i \in R^S$ are large vectors, hence, the *a* and *s* subscripts are ignored here). Then we can state the payoff function of player *i* as follows

(10) $$\Pi_i(k) = \Pi_i(k_i, k_{-i}) = \max_{e_i} L_i(n^*, e_i, e^*_{-i}, y^*, k)$$

where

(11) $$L_i(n, e_i, e_{-i}, y, k) = \sum_{s=1}^{S} \delta_i^s (r_{i,s} - k_i \psi_{i,s}) + y\Phi^-(n, e_i, e_{-i}, k)$$

is a modified Lagrangian, y is the Lagrange multiplier, (e^*_i, n^*, y^*) are equilibrium solutions of the variables in question, and Φ^- is given by $\min(0, \Phi)$. The adjustment rules in the algorithm are then given by

(12) $$\dot{e}_i = \frac{\partial L_i(n, e_i, e_{-i}, y, k)}{\partial e_i}, \quad \forall i$$

(13) $$\dot{y} = -\frac{\partial L_i(n, e_i, e_{-i}, y, k)}{\partial y} = -\Phi^-$$

(14) $$\dot{n} = y\frac{\partial \Phi^-}{\partial n}$$

where $\frac{\partial L_i(.)}{\partial e_i}$ and $\frac{\partial L_i(.)}{\partial y}$ are the partial derivatives of $L_i(.)$ with respect to e_i and y, and $\frac{\partial \Phi^-}{\partial n}$ is the partial derivative of the constraint function with respect to *n*.

The algorithm then comes in differential form: Starting at arbitrary initial points (e_i, y, n), the dynamics represented by the adjustment rules are pursued all the way to the stationary points (e^*_i, n^*, y^*). Such points satisfy, by definition, the steady-state generalized equation system:

$$0 = \Phi^-(n, e_i, e_{-i}, k),$$

$$0 \in \partial_{e_i}\left[\sum_{1}^{S} \delta_i^s (r_{i,s} - k_i \psi_{i,s}) + y\Phi^-(n, e_i, e_{-i}, k)\right], \quad \forall i$$

with $y^* \geq 0$. It is standard from mathematical programming that individual optimality and stock feasibility then obtains. The numerical scheme uses Euler's method to integrate (12), (13), and (14) all the way to the equilibrium solutions $\left(e_i^*, n^*, y^*\right)$.

5.0 Numerical results

A strategic form for our game is given in table 5.1. To obtain these results, the newly developed dynamic simulation software package, POWERSIM[11], is used as computational support. The parameter values listed in table 5.0 are used for the computations. In addition, α and γ are set equal to 1.01 and 1.5, respectively, to give a maximum biomass of 6 million tonnes for a pristine stock. Based on the survival rate of cod, ξ_a is given a value of 0.81. The price parameter $V_,$ is set equal to NOK 6.78. The variable costs (ϑ_T & ϑ_C) and fixed costs (φ_T & φ_C) for engaging a vessel are calculated to be (NOK 12.88 & 0.88) and (NOK 15.12 & 0.65) million for T and C, respectively[12]. The interest rate, r_i, is set equal to 7% as recommended by the Ministry of Finance of Norway. The initial number of cod of each age group is calibrated using the 1992 estimate of the stock size of cod in tonnes[13].

[Table 5.0 in here!]

In table 5.1, rows represent player T's pure strategies k_T and columns represent player C's pure strategies k_C. Player T's payoff is placed in the southwest corner of the cell in a given row and column and the payoff to player C is placed in the northeast corner. The best payoff for player T in each column and the best payoff to player C in each row are bold-faced. As an example notice that 19.4 has been bold-faced in cell k_T = 65 and k_C = 500, since 19.4 lies in row k_T = 65, this tells us that pure strategy k_T = 65 is player T's best reply to a choice of pure strategy k_C = 500 by player C.

[Table 5.1 in here!]

Notice that the only cell in table 5.1 that has both payoffs bold-faced is that which lies in row k_T = 57 and column k_C = 1050. Thus the only pure strategy pair that constitutes a Nash equilibrium is (57,1050). Each strategy in this pair is a best reply to the other. This gives total PV of economic benefit equal to $\sum_i \Pi_i(57{,}1050)$ = NOK 25.87 billion, with $\Pi_T(57{,}1050)$ = NOK 11.84 billion and $\Pi_C(57{,}1050)$ = NOK 14.03 billion, respectively.

[Table 5.2 in here!]

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[11] Powersim is a dynamic simulation software package developed by ModellData AS in Bergen, Norway. The model has many powerful features, including the ability to process array variables.

[12] The price per kilogram of NOK 6.78 is taken from table 22 in Central Bureau of Statistics of Norway (1989-1990). The parameters ϑ_i and φ_i are calculated using cost data in Lønnsomhetsundersøkelser (1979 - 1990).

[13] The 1992 stock size is estimated at 1.8 million tonnes (Ressursoversikt, 1993).

The overall PV of economic rents from the fishery as a function of k_T and k_C are given in table 5.2. The entries in each cell of this table are simply the sum of the entries in corresponding cells in table 5.1. These results indicate an optimal fleet consisting of 1100 coastal vessels and no trawlers with PV of economic rent equal to NOK 36.11 billion, or NOK 32.83 million per vessel. In contrast, a discounted profit maximising fleet consisting solely of trawlers would be made up of 70 vessels and earn a PV of economic rent of NOK 32.42 billion, or NOK 463.14 million per vessel.

The model thus appears to support the general theoretical assertion that non-cooperation generally results in rent dissipation through the use of excess fishing capacity. The economic theory of fisheries predicts that in an open access fishery, economic rent would normally be dissipated completely (Gordon, 1954; Hannesson, 1993). Table 5.2 indicates that a trawl-coastal fishery vessel combination of a little over 120-2500 vessels would dissipate discounted economic rent from the fishery to nil. Thus, this vessel combination or its equivalent, is our models prediction of the open access fishing capacity. If the agents in the fishery were to receive subsidies totally NOK 9.48 billion (in present value) in addition to having open access to the resource, then the model's prediction of fishing capacity is 140 trawlers and 3000 coastal vessels or their equivalent. It should be interesting to compare the equilibrium stock and harvest profiles that would result under "open access plus subsidy", open access, Nash non-cooperative, and the sole ownership equilibria. This is done graphically in figures 5.1 and 5.2 below. The figures illustrate clearly the adverse effects of "open access plus subsidy", open access, Nash non-cooperative equilibria as compared to the optimal solution.

[Figures 5.1 and 5.2 in here!]

Notice that contrary to expectations, the biomass is not completely depleted at the end of the game. There are two possible reasons for this. In the first place players are not allowed to exploit all age groups of fish. Secondly, even if this was allowed, it would not be economically profitable to catch every single fish available.

5.1 Perfect malleability versus perfect non-malleability

We label the model in Sumaila (1994) the perfect malleable capital model and that in this paper the perfect non-malleable capital model. The purpose here is to compare the capacity investments and the PV of economic rent accruable to the players both individually and to society at large, in the two models. To do this, the perfect malleable model is run using comparable prices and costs. Table 5.3 below gives the capacity predictions of the two models, and the PV of economic rents to the players when both agents are active; when only T is active; and when only C is active. It is seen from this table that vessel capacity investment varies from year to year in the case of the malleable capital model.

[Table 5.3 in here!]

A possible interpretation here is that each player evaluates his optimal capacity requirement in a given year and then rents precisely this quantity from a rental firm for fishing vessels[14]. For instance, when both are active, T's optimal vessel size varies from a high of 65 in the third year to a low of 41 trawlers in year 15, while C's varies from a high of 939 in year 3 to a low of 564 in the last year. In the non-malleable capital model, however, T's *ex ante* fixed capacity investment is 57 trawlers, and the corresponding capacity investment for C is 1050 coastal vessels.

The economic results given by the two models are given in the third column of table 5.3. Two important observations can be made from this table. First, the maximum economic rent from the resource are different in the two models: NOK 44.53 billion is achieved in the malleable capital model and NOK 36.11 in the non-malleable one. The higher PV of economic rent achievable in the malleable capital model can be attributed to the removal of the restriction that non-malleability of capital imposes on the agents. The negative impact of this restriction is quite substantial, reaching upto about NOK 12 billion (or 47% of what is achievable under the restriction) in the case of the Nash equilibrium solution. This clearly demonstrates that there is much to be gained from establishing rental firms for fishing vessels. Or rather, by allowing mobility of vessels between different stocks[15].

Second, in both models the best economic results are achieved when C operates the fishery alone. However, the superiority of C becomes sharper in the non-malleable capital model: There is a difference of well over 10% (in favour of C) in the PV of economic rent accruable in the non-malleable capital model. In the malleable capital model, however, a difference of only about 5% is noted. This finding may have to do with the relatively high fixed cost of trawlers and the fact that fixed costs must be taken as given by the players in the non-malleable capital model.

5.2 Comparison with available data on the Arcto-Norwegian cod fishery

It is stated in section 2 that in 1991 the equivalent of fishing capacity of about 638 coastal vessels and 58 trawlers were operated by Norwegian fishers to land 130 and 270 thousand tonnes of cod, respectively. This implies that catch per trawler is about 4655 tonnes and catch per coastal fishery vessel is 203 tonnes. Now, the Nash equilibrium strategies stipulate vessel sizes of 1050 for C and 57 for T. Together, these capacities are used to land an average of about 842 thousand tonnes a year[16]. Of the total, trawlers land 412 thousand tonnes and the coastal fishery vessels 430 thousand tonnes. Thus catch per vessel are 7228 and 410 tonnes for a trawler and coastal fishery vessel, respectively. These numbers signify the incidence of overcapacity in the Arcto-Norwegian cod fishery even in comparison to the results from a non-cooperative solution. Comparison with the sole owner's optimal solution reveals an even greater degree of overcapacity in the fishery: In this case 1100 vessels are used to land on average about 770 thousand tonnes of fish per year by C (that is, 700 tonnes per vessel) and 70 trawlers to land an average of about 780 thousand

[14] A practical way to view these variations is that the agents in this model have alternative uses for their vessel capacities, thereby making it possible for them to divert excess capacity in any given year to such uses.

[15] Hannesson (1993) looks more closely at the possible gains from allowing mobility of vessels between different stocks.

[16] Note that these harvests comprise both Norwegian and Russian landings, since we do not differentiate between the two in our model.

tonnes per year by T (well over 10000 tonnes per vessel). A catch per trawl vessel of 10000 tonnes per year appears to be high, probably the proportionality assumption (underlying the harvest function) between the stock size and the catch per vessel is appropriate only when variation in the stock size is not too large.

5.3 Effect of fixed costs, interest rates, initial stock size, and terminal constraint

Fixed costs Sensitivity analysis shows that (for the game solution), the elasticity of the PV of economic benefits accruable to the players T and C with respect to fixed costs are about -0.7 and -0.4, respectively[17]. Hence, to achieve the higher discounted economic rent, T needs a drop in its fixed costs relative to those of C of about 28%. The equivalent elasticities in the sole owner solutions are -0.3 for T and -0.2 for C, which implies that T needs a drop in its fixed costs relative to those of C of about 39% to take over from C as the producer of the optimal solution. We also investigated the effects of zero fixed costs. This is the same as assuming that fixed costs are considered to be "sunk" by the agents. Under such an assumption, C and T achieve discounted economic rents of NOK 20.27 and 19.7 billion, respectively, in the game situation, and NOK 42.06 (C) and 42.64 (T) in the sole ownership solutions. We see that the previously clear superiority of C is now neutralised to a great extent.

Interest rates For the game situation, we found that a 1% drop in the interest rate faced by both players results in a 1% increase in the relative profitability[18] of T. On the other hand, an equivalent drop in the interest rate in the sole ownership scenario leads to an increase of 0.5% in the relative profitability of C. Intuitively, it is not difficult to understand why T does relatively better in the game situation while C does relatively better in the sole ownership case: Since T harvests everything from age group 4 and above, while C harvests only age groups 7 and above, it is no wonder that T is the one best positioned to capitalize on the increase in patience that a decrease in interest rate entails. C does better in the sole ownership scenario because an increase in patience plus the fact that C harvests fish from age group 7 and above means that a larger proportion of the stock will reach maximum weight before they are harvested, thereby resulting in better relative profitability for C.

Initial stock size To investigate the effect of the initial stock size on the relative profitability of the agents, the model is re-run with 50% and 150% of the base stock size of 1.8 million tonnes. The results we obtained indicate that in the sole ownership solutions, T improves it's relative profitability as the stock size increases; from 86.85% when the stock size is only 50% of the base case to 92.1% when the stock size is 150% of the base case. The effect of stock size on the relative profitability of the agents in the game solution is however not that clear: T increases it's relative profitability both when the stock size is only 50% of the original (90.6% as against 84.4% in the base case) and when the stock size is 150% of the base stock size. In this case T's relative profitability is 92.3%. As these numbers show T's relative

[17] Recall that the elasticity of a function, f(x,y), with respect to x is defined as the percentage increase in f(x,y) resulting from a 1% increase in x.

[18] Relative profitability is defined as discounted economic rent to T divided by discounted economic rent to C multiplied by 100.

profitability increases by a larger margin when the stock size increases than when it decreases.

Terminal constraint on the stock size to be left behind at the end of the game A requirement that not less that 50% of the initial stock size should be left in the sea at the end of the game changes the outcome of the game significantly in the game situation. In the sole ownerhip situation, however, the same solutions as in the base case are obtained, mainly because this constraint is not binding in these cases. Under such a requirement it turns out that T comes out better (in contrast to the base case where it is C that does better) , earning NOK 15.97 billion as against C's NOK 12.98 billion. An important point to note here is that the introduction of the terminal constraint, in the game environment, leads to an increase in the overall benefit from the fishery from NOK 25.87 to NOK 28.95 billion. This explains why economists advocate regulation when common property resources are exploited in non-cooperative environments.

6.0 Concluding remarks

The main findings of this study can be stated as follows: The optimal capacity investments in terms of number of vessels for T and C in a competitive, non-cooperative environment are 57 trawlers and 1050 coastal vessels, respectively. The use of these capacities results in discounted benefits of NOK 11.84 and 14.03 billion, respectively, to T and C, and an overall discounted economic benefit of NOK 25.85 billion to society at large. Using only T and C vessels in the exploitation of the resource, the optimal fleet sizes are 70 and 1100, respectively. In these cases the PV of economic rents are NOK 32.42 and 36.11 for T and C. We also found out that, as expected, the results obtained are rather sensitive to perturbations in fixed costs, interest rates, initial stock size and the terminal constraint.

It is in order to state here that given that modeling and computation are always exercises in successive approximation (Clark & Kirkwood, 1979), our estimates should not be taken too literally. Having said this, the results of the analysis indicate that in it's current state the Arcto-Norwegian cod fishery appears to suffer from over capacity. Hence, the practical implication of this study with respect to efficient management of the resource is that the excess capacity should be run down as rapidly as possible to a certain level[19] somewhere above the Nash equilibrium capacity level. Thereafter the remaining excess capacity is allowed to depreciate to the "desired" level by itself. From then on new capacity investment is undertaken only to make up for depreciation. This would ensure each player his best possible outcome and the society the second best solution.

The practical implication of the results obtained would have been somewhat different if the fishery were under-exploited. In this case, we distinguish between starting a completely new fishery and the case where fishing is currently in progress. In the case of a new fishery, there is a real possibility for realising the first best solution by allowing only C to exploit the resource with a capacity size of about 1100 coastal

[19] The choice of this level will depend on both the depreciation rate and the difference between the cost of acquiring a new vessel and the disinvestment resale price of the vessels relative to the price of new vessels.

fishery vessels. However, if political, social and cultural realities dictate the participation of both players and this were to be in a non-cooperative environment, then each player should aspire to start off with its Nash equilibrium capacity size as computed herein.

Appendix

The algorithm. The following non-standard Lagrangian function for player *i* follows from our model:

$$L_i(n,e,y,k) = \sum_{s=1}^{S} \left[\begin{array}{l} \delta_i^s (r_{i,s} - k_i \psi_{i,s}) + y_{0,s}(f(B_{s-1}) - n_{0,s})^- \\ + y_{A,s}(\xi_A n_{A,s-1} + \xi_{A-1} n_{A-1,s-1} - n_{A,s} - h_{A,s})^- \\ + \sum_{a=1}^{A-1} y_{a,t}(\xi_{a-1} n_{a-1,s-1} - n_{a,s} - h_{a,s})^- \end{array} \right]$$

where y := $y_{a,s}$ is the player-invariant, but age and season-variant multipliers, and all other variables are as defined earlier.

The negative superscript on some components of the equation above is a device introduced to ensure monotone convergence of multipliers by focusing attention on the situations where there are constraint violations. Such a device results in multipliers that are different from those associated with the classical Lagrangians. There is a relationship between the two kinds of multipliers, however, the exact relationships are not so easy to retrieve. It is therefore necessary, at this juncture, to call for caution when interpreting the computed equilibrium multiplier levels.

The gradient information obtainable from $L_i(n,e,y)$, gives the adjustment equations for the effort levels and multipliers. We first introduce a special (switch) function related to the derivative of Φ^-, before we state the adjustment equations. Let the function $H(r) = 1$ if r<0, and $H(r) = 0$ otherwise. If $r \geq 0$ were a constraint inequality, then H(r) will attain a value of 1 if the constraint is violated, otherwise it attains a value of 0. In writing the adjustment equations below, this switch function is used.

Starting at arbitrary initial guesses of $y_{a,t}$, $n_{a,t}$ and $e_{i,t}$, we pursue the dynamics given by the adjustment equations below, all the way to the equilibrium solutions.

Effort adjustment. The adjustment equation for effort given by $\dfrac{\partial L_i(.)}{\partial e_{i,s}}$ is

$$\begin{aligned} e_{i,s} = {} & \delta_i^s \Big(\sum_a V_i w_a q_{i,a} k_i n_{a,s} - k_i \vartheta_i e_{i,s}^b\Big) \\ & + \sum_{a=1}^{A-1} y_{a,s} H(\xi_{a-1} n_{a-1,s-1} - n_{a,s} - h_{a,s})(-q_{i,a} k_i n_{a,s}) \\ & + y_{A,s} H(\xi_A n_{A,s-1} + \xi_{A-1} n_{A-1,s-1} - n_{A,s} - h_{A,s})(-q_{i,A} k_i n_{A,s}) \end{aligned}$$

Multiplier adjustment. The equations for the sequential adjustment of the multipliers are obtained by taking the negative of the partial differential of L_i with respect to the appropriate multiplier, that is, they come from $-\dfrac{\partial L_i(.)}{\partial y_{a,s}}$.

For age group zero fish, the multiplier is adjusted according to the equation

$$\dot{y}_{0,s} = -H(f(B_{s-1}) - n_{0,s})(f(B_{s-1}) - n_{0,s})$$

Multipliers for fish of age groups between 1 and A-1 are adjusted as follows

$$\dot{y}_{a,s} = -H(\xi_{a-1}n_{a-1,s-1} - n_{a,s} - h_{a,s})(\xi_{a-1}n_{a-1,s-1} - n_{a,s} - h_{a,s})$$

For the last age group, multiplier adjustment is according to

$$\dot{y}_{A,s} = -H(\xi_A n_{A,s-1} + \xi_{A-1}n_{A-1,s-1} - n_{A,s} - h_{A,s})(\xi_A n_{A,s-1} + \xi_{A-1}n_{A-1,s-1} - n_{A,s} - h_{A,s})$$

Here, the RHS of the equations are calculated and then the corresponding multipliers adjusted according to the magnitude and direction of the calculated result.

Nature's adjustment of the stock level. Natures objective can be expressed as

$$\begin{aligned} L_N &= y_{0,s}\left(f(B_{s-1}) - n_{0,s}\right)^- \\ &+ \sum_{a=1}^{A-1} y_{a,s}(\xi_{a-1}n_{a-1,s-1} - n_{a,s} - h_{a,s})^- \\ &+ y_{A,s}(\xi_A n_{A,s-1} + \xi_{A-1}n_{A-1,s-1} - n_{A,s} - h_{A,s})^- \end{aligned}$$

This equation is derived from the fact that once the stock dynamics is obeyed, nature's net benefit is 0, hence, L_N consist of only the constraint equations.

The updating rules for age groups, a=0, a=1,...,A-2, a=A-1, and a=A are different and are given below separately. These are obtained by partially differentiating $L_{N}(.)$ with respect to the corresponding stock level. That is, they come from $\frac{\partial L_N(.)}{\partial n_{ai,s}}$.

(1) The stock level of age zero fish is adjusted sequentially in accordance with the equation,

$$\begin{aligned} \dot{n}_{0,s} &= y_{0,s+1}H(f(B_s) - n_{0,s+1})f'(B_s)\frac{\partial B_s}{\partial n_{0,s}} \\ &- y_{0,s}H(f(B_{s-1}) - n_{0,s}) \\ &+ y_{1,s+1}H(\xi_0 n_{0,t} - n_{1,s+1} - h_{1,s+1})\xi_0 \end{aligned}$$

(2) Fish of age groups between 1 and A-2 are updated as follows,

$$\begin{aligned}\dot{n}_{a,s} &= y_{0,s+1}H(f(B_s) - n_{0,s+1})f'(B_s)\frac{\partial B_s}{\partial n_{a,s}} \\ &+ y_{a,s}H(\xi_{a-1}n_{a-1,s-1} - n_{a,s} - h_{a,s})(-1 - \sum_i q_{i,a}k_i e_{i,s}) \\ &+ y_{a+1,s+1}H(\xi_a n_{a,s} - n_{a+1,s+1} - h_{a+1,s+1})\xi_a\end{aligned}$$

(3) The last but one age group of fish (i.e., the A-1 age group) is adjusted in accordance to the equation,

$$\begin{aligned}\dot{n}_{A-1,s} &= y_{0,s+1}H(f(B_s) - n_{0,s+1})f'(B_s)\frac{\partial B_s}{\partial n_{A-1,s}} \\ &+ y_{A-1,s}H(\xi_{A-2}n_{A-2,s-1} - n_{A-1,s} - h_{A-1,s})(-1 - \sum_p q_{i,A-1}k_i e_{i,s}) \\ &+ y_{A,s+1}H(\xi_A n_{A,s} + \xi_{A-1}n_{A-1,s} - n_{A,s+1} - h_{A,s+1})\xi_{A-1}\end{aligned}$$

(4) Finally, the last age group, is updated using the following equation,

$$\begin{aligned}\dot{n}_{A,s} &= y_{0,s+1}H(f(B_s) - n_{0,s+1})f'(B_s)\frac{\partial B_s}{\partial n_{A,s}} \\ &+ y_{A,s+1}H(\xi_A n_{A,s} + \xi_{A-1}n_{A-1,s} - n_{A,s+1} - h_{A,s+1})\xi_A \\ &+ y_{A,s}H(\xi_A n_{A,s-1} + \xi_{A-1}n_{A-1,s-1} - n_{A,s} - h_{A,s})(-1 - \sum_i q_{i,A}k_i e_{i,s})\end{aligned}$$

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Table 2.1: No. of Norwegian vessels operating on the "cod fishes group" for five different years.

Year	Trawlers	Coastal vessels
1991	57	562
1990	51	572
1988	84	661
1986	118	628
1984	128	718

Table 5.0: Values of parameters used in the model

Age a (years)	Selectivity q(p,a) p=1	p=2	Weight at spawning w(s,a) (kg)	Weight in catch w(a) (kg)	Initial numbers (millions)
0	0	0	0.090	0.10	167.0
1	0	0	0.270	0.30	135.0
2	0	0	0.540	0,6	108.0
3	0	0	0.900	1.00	88.3
4	0.0074	0	1.260	1.40	71.7
5	0.0074	0	1.647	1.83	58.3
6	0.0074	0	2.034	2.26	46.7
7	0.0074	0.00593	2.943	3.27	38.3
8	0.0074	0.00593	3.843	4.27	30.8
9	0.0074	0.00593	5.202	5.78	0,25
10	0.0074	0.00593	7.164	7.96	20.3
11	0.0074	0.00593	8.811	9.79	16.7
12	0.0074	0.00593	1.0377	11.53	13.3
13	0.0074	0.00593	12.456	13.84	10.8
14	0.0074	0.00593	13.716	15.24	8.67
15	0.0074	0.00593	14.706	16.34	7.0

Note: (1) The values for q(p,a) are calculated using the procedure outlined in Sumaila (1994). (2) Player T exploits fish of age 4 and above and player C fish of age group 7 and above (Hannesson, 1993).

Table 5.1: The bimarix game. Gives the payoff to each player as a function of k1 (no. of T vessels) and k2 (no. of C vessels) in billions of NOK. Player T's payoff is placed in the southeast corner of the cell in a given row and column, and the payoff to player C is placed in the northeast corner.

k1	k2 (No. of C vessels)											
(No. of T vessels)	0	500	700	900	1000	1050	1100	1200	1500	2000	2500	3000
0		28,22 0	31,76 0	35,34 0	35,99 0	35,79 0	**36,11** 0	35,3 0	33,89 0	28,63 0	26,67 0	20,87 0
20	0 17,52	21,28 13,3	24,58 11,32	**25,53** 9,51	25,28 8,67	25,49 8,38	25,44 8,09	25,04 7,46	23,99 6,08	24,21 4,31	18,5 3,54	14,07 2,66
40	0 28,72	15,54 18,8	17,1 15,29	17,7 12,75	17,67 11,65	**18,13** 11,39	17,99 10,92	17,89 10,07	17,12 8,14	14,47 5,64	13,3 **4,48**	9,85 **3,19**
45	0 29,64	14,18 19,1	15,66 15,57	16,3 13,02	16,28 11,9	16,54 11,57	**16,58** 11,11	**16,58** 10,35	15,86 8,22	13,94 **5,82**	12,08 4,36	8,6 2,97
50	0 30,57	12,92 19,25	14,48 15,89	15,02 13,23	15,09 12,04	**15,42** 11,78	15,28 11,19	15,4 **10,47**	14,66 8,23	12,99 5,74	11,4 4,27	7,54 2,66
55	0 31	11,84 19,22	13,35 15,89	14,01 13,25	14,1 12,12	14,34 11,8	**14,36** 11,29	14,22 10,4	13,64 8,24	11,8 5,6	10,95 4,22	6,81 2,45
57	0 31,45	11,48 19,2	12,98 15,97	13,66 12,1	13,66 12,1	**14,03** **11,84**	13,99 **11,37**	13,76 10,36	13,54 **8,32**	11,73 5,59	10,46 4,34	6,45 2,26
60	0 31,57	10,87 19,08	12,43 15,96	13,18 **13,42**	13,24 12,2	13,26 11,67	**13,42** 11,31	13,33 10,36	12,89 8,19	11,25 5,46	9,7 3,81	5,81 2,15
65	0 32,17	10,29 **19,4**	11,74 **16**	12,3 13,3	12,7 **12,31**	12,67 11,74	12,66 11,29	**12,75** 10,42	12,11 8,04	10,81 5,35	8,81 3,6	6,01 1,95
70	0 **32,42**	9,41 18,9	10,82 15,65	11,52 13,13	11,58 11,86	11,92 11,63	**11,95** 11,2	**11,95** 10,24	11,72 8,04	9,83 4,92	8,3 3,38	4,77 1,41
80	0 31,7	8,13 18,2	9,46 15,06	**9,88** 12,24	10,26 11,39	**10,67** 11,34	10,56 10,67	**10,67** 9,83	10,39 7,54	8,24 4,17	7,47 2,71	3,58 0,54
90	0 30,05	7,33 17,7	8,28 14,17	9,11 12,02	9,4 11,12	9,33 10,45	**9,63** 10,32	9,12 8,82	9,35 6,97	7,16 3,41	6,31 1,94	2,03 -0,63
100	0 28,23	6,48 16,69	7,63 13,84	8,2 11,41	8,64 10,57	**8,88** 10,28	8,53 9,33	8,66 8,61	8,37 6,08	6,43 2,51	4,93 0,69	0,97 -1,65
120	0 23,5	5,02 14,03	5,91 11,24	6,48 9,1	6,36 7,73	**6,56** 7,38	6,51 6,84	6,35 5,79	6,11 3,62	4,09 0,15	2,57 -1,84	-0,69 -3,89
140	0 21,63	4,42 12,89	5,35 10,42	5,66 7,81	5,41 5,99	**5,98** 5,66	5,52 5,08	5,59 4,42	5,16 1,99	2,59 -2,18	0,17 -4,58	-2,74 -6,73

Table 5.2: Overall PV of economic rent from the fishery as a function of k1 (no. of T vessels) and k2 (no. of C vessels), in billions of NOK.

k1	k2 (No. of CFM vessels)										
No. of TF vessels	0	500	700	900	1000	1050	1100	1200	1500	2000	2500	3000
0		28,2	31,8	35,3	36	35,8	36,1	35,3	33,9	28,6	26,7	20,87
20	17,5	34,6	35,9	35	34	33,9	33,5	32,5	30,1	28,5	22,1	16,73
40	28,7	34,3	35,4	30,5	29,3	29,5	28,9	27,9	25,3	20,1	17,8	13,04
45	29,6	33,3	31,2	29,3	28,2	28,1	27,7	26,9	24,1	19,8	16,4	11,57
50	30,6	32,2	30,4	28,2	27,1	27,2	26,5	25,9	22,9	18,7	15,6	10,2
55	31	31,1	29,2	27,3	26,2	26,1	25,7	24,6	21,9	17,4	15,2	9,26
57	31,4	30,7	29	26,8	25,8	25,9	25,4	24,1	21,8	17,3	14,8	8,71
60	31,6	30	28,4	26,6	25,4	24,9	24,7	23,7	21,1	16,7	13,5	7,95
65	32,2	29,7	27,8	25,6	25	24,4	24	23,2	20,2	16,2	12,4	7,96
70	32,4	28,3	26,5	24,6	23,4	23,6	23,2	22,2	19,8	14,7	11,7	6,18
80	31,7	26,3	24,5	22,1	21,7	22	21,2	20,5	17,9	12,4	10,2	4,13
90	30	25	22,5	21,1	20,5	19,8	20	17,9	16,3	10,6	8,3	1,4
100	28,2	23	21,5	19,6	19,2	19,2	17,9	17,3	14,5	8,9	5,6	-0,68
120	23,5	19,1	17,2	15,6	14,1	13,9	13,4	12,1	9,7	4,2	0,73	-4,59
140	21,6	17,3	15,8	13,5	11,4	11,6	10,5	10	7,2	0,4	-4,4	-9,48

Table 5.3: Malleable vs. Non-malleable capital. Gives the equilibrium vessel sizes and the overall discounted economic rent that accrues to society from the resource.

	Vessel size (in numbers)		PV of economic benefit (in billion NOK)	
	Malleable	Non-malleable	Malleable	Non-malleable
Both active	(65 -41; 939 - 564)	(57;1050)	38.26	25.87
T active	(80 -49)	(70)	42.35	32.42
C active	(1153 - 761)	(1100)	44.53	36.11

Fig 5.1: Stock profiles (in million tonnes): Illustrates the post-catch stock size in each period for open access plus subsidy (OAS), open access (OA), Nash equilibrium (NE), T only and C only (the optimal solution).

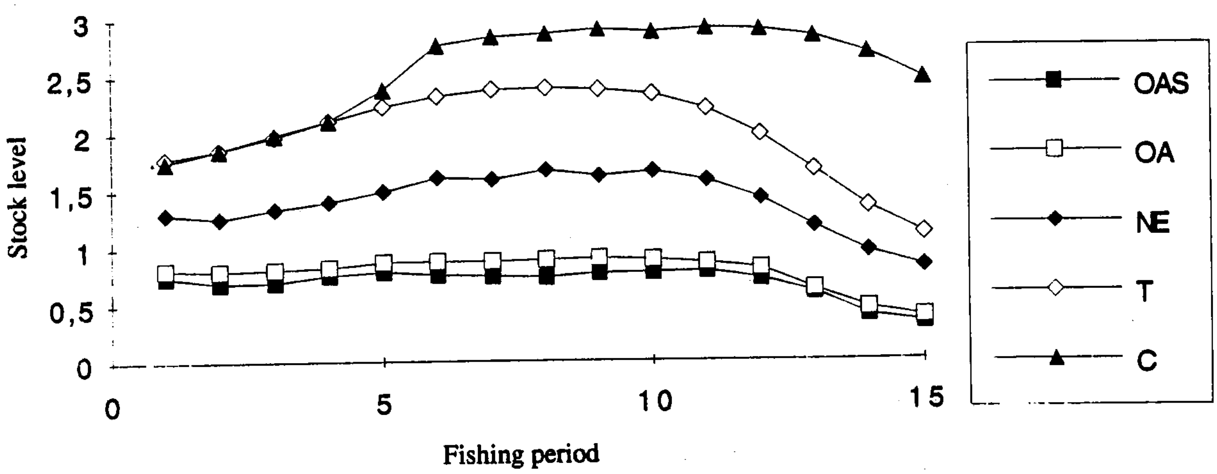
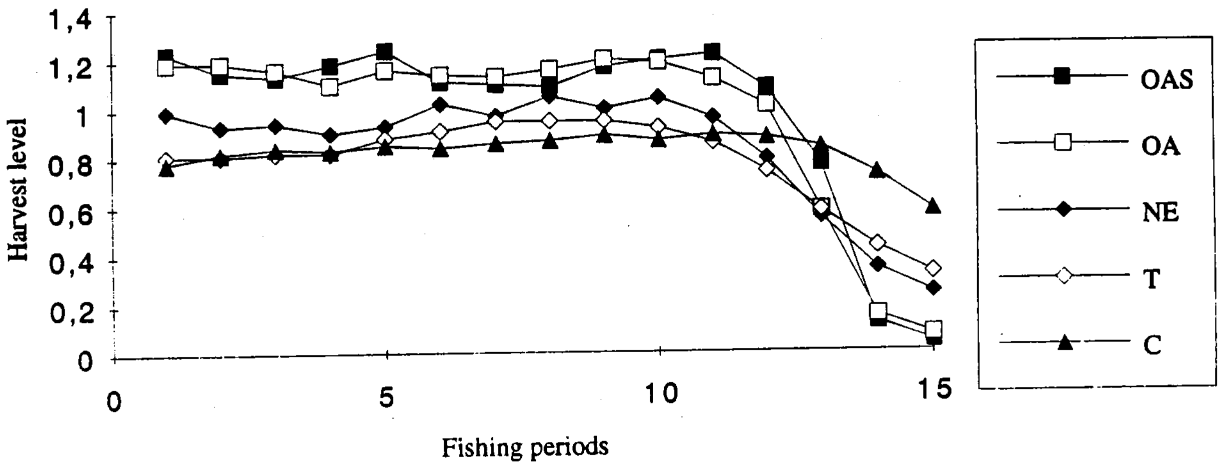


Fig. 5.2: Harvest profiles (in million tonnes): Illustrates total harvest in each period for open access plus subsidy (OAS), open access (OA), Nash equilibrium (NE), T only and C only (the optimal solution).



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Non-Cooperation in Fish Exploitation

The Case of Irreversible Capital Investment in the Arcto-Norwegian Cod Fishery

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Working Paper
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ISSN 0804-3639

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Bergen, December 1994, 26 pp.

Summary:

A two-stage, two-player non-cooperative game model is developed under an irreversible capital investment assumption. The main aim is to predict the number of vessels that each player in such a game will find in his best interest to employ in the exploitation of the Arcto-Norwegian cod stock, given a non-cooperative environment and the fact that all players are jointly constrained by the population dynamics of the resource. The predictions obtained are then compared with (i) the sole owner's optimal capacity investments for the two players; (ii) the results in Sumaila (1994), where perfect malleability of capacity is assumed implicitly; and (iii) available data on the Arcto-Norwegian cod fishery.

Sammendrag:

Et to-steg to-aktør ikke-kooperativt spill er utviklet under en antagelse om irreversibel kapital investering. Hovedmålet er å finne antall fiskebåter som hver aktør finner i sin egen interesse å sette inn for å høste av den norsk arktiske torsken, gitt begrensninger i ressurstilgangen og at aktørene ikke samarbeider. Resultatene som framkommer er sammenliknet med (1) resultatet når bare en av aktørene har rettigheten til ressursen, (2) resultatene i Sumaila (1994) der man har en reversibel kapital antagelse, og (3) tilgjengelig data på de norsk arktiske torskefiskeriene.

Indexing terms:	**Stikkord:**
Game theory	Spillteori
Fishery	Fiske
Fishery resources	Fiskeressurser
Coastal vessels	Kystbåter
Cod	Torsk
Trawlers	Trål
Norway	Norge

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1.0 Introduction

This paper considers non-cooperative use of a common property fish stock, namely the Arcto-Norwegian cod. Attention is focused on a restricted access fishery where only two-agents participate in the exploitation of the resource, the aim being to predict the number of vessels that each agent in such a situation will find in his best interest to employ. An important although self-evident aspect of the game is that both agents are jointly constrained by the population dynamics of the resource. The key assumption of the paper is that players undertake investment in capital that is irreversible. This assumption is quite realistic because capital embodied in fishing vessels is often non-malleable: Non-malleability is used here to refer to the existence of constraints upon the disinvestment of capital assets utilized in the exploitation of the resource (Clark et al. 1979). This implies that once a fishing firm or authority invests in a fleet of vessels it either has to keep it until the fleet is depreciated, or else the vessels can only be disposed off at considerable economic loss.

A number of papers have appeared in the fishery economics literature that focus, among other things, on the irreversibility of capital employed in the exploitation of fishery resources. Examples include Clark et al. (1979), Clark & Kirkwood (1979), Dudley & Waugh (1980), Charles (1983a, 1983b), and Charles & Munro (1985). We are, however, not aware of any prior work that models, computes numerically and analyses the exploitation of fishery resources as done in this paper. Among the examples cited above, only Dudley & Waugh (1980) consider investment decision in a fishery with more than a single agent participating. But even in this case, only qualitative statements of the likely effects of this are made. The study of Clark & Kirkwood (1979) is close to the work planned herein, at least in terms of the kind of questions they address. The authors presented a bioeconomic model that predicts the number of vessels of each of the two types entering the prawn fishery of the Gulf of Carpentaria under free access. In addition, they estimated the economically optimal number of vessels of each type. The results they obtained are then compared with available data on the prawn fishery of the Gulf of Carpentaria.

These are issues we also address here albeit with a number of differences. First, there is a difference with respect to the number of agents in the two studies: While Clark & Kirkwood (1979) consider the social planner's and open access equilibrium fleet sizes, we compute equilibrium fleet sizes that will emerge in a non-cooperative environment involving two agents, and then, using these results, we derive the social planner's equilibrium fleet size and discuss the probable open access equilibrium fishing capacity. Thus, we add a new dimension to the discussion, namely, the two-agent analysis[1]. Second, there is a difference in the way we model the population dynamics of the fish stock: While their study prescribes and uses a single cohort to describe the fish stock, we accommodate a multicohort population structure.

The primary concern of this study is to develop the necessary framework to

(1) identify a Nash non-cooperative equilibrium solution for a bimatrix game involving the trawl and coastal fisheries operating on the Arcto-Norwegian cod;

---

[1] The motivation for undertaking a two-agent analysis is given in section 2 of this paper.

(2) identify the sole owner equilibrium solutions for the two fisheries, and determine which among these gives the optimal solution;

(3) compare the results in (1) and (2) above to (i) the results in Sumaila (1994), where perfect malleability of capital is assumed implicitly, and (ii) with available data on the Arcto-Norwegian cod. The former comparison would put us in a position to say something about the possible gains of establishing rental firms for fishing vessels and/or allowing mobility of vessels between different stocks;

(4) discuss the fishing capacities that are likely to emerge in an open access scenario; and

(5) investigate the effect of fixed cost, interest rates, initial stock size, and the terminal constraint, on the relative profitability of the players.

The next section gives a brief description of the Arcto-Norwegian cod fishery. Section 3 presents the model, a special feature of which is the explicit modeling of the biologically and economically important age groups of cod. This is followed by a brief mention of the algorithm for the computation of the equilibrium solutions: The detailed algorithm is relegated to an appendix. In section 5, the results of the study are stated. Finally, section 6 concludes the paper.

2.0 The Arcto-Norwegian cod fishery

The Arcto-Norwegian cod, *gadus morhua,* is a member of the Atlantic cod family, arguably among the world's most important fish species. It inhabits the continental shelf from shoreline to 600 m depth, or even deeper, usually 150-200 m. It is gregarious in behaviour, forming shoals or schools and undertaking spawning and feeding migrations. The diet of adult cod is variable and consists mainly of herring, capelin, haddock and codling. The Arcto-Norwegian cod spawns only along the Norwegian coast, mainly in Lofoten in April-March. Typically, it starts spawning at the age of 7-8 years; eggs are carried by the gulf stream, over the coast where they hatch, and into the Barents Sea, up towards Svalbard, where the young cod grow. It has a relatively long life span: it can live for well over 15 years. A majority of young cod die quite early, either because of a lack of adequate food, or because they are eaten up by other fishes. Young cod between the ages of 3-6 come to the Finnmark's coast every year. This is because mature capelin, which cod preys on, move to their spawning spots close to the Finnmark's coast. Cod follows and predates them, thus resulting in good spring cod in the period April to June.

The Arcto-Norwegian cod is a shared resource jointly managed by Norway and Russia. Norwegian fishers employ mainly coastal and trawl fishery vessels in the exploitation of the resource, while their Russian counterparts employ mainly trawlers. Table 2.1 gives the number of Norwegian trawl and coastal fishery vessels (of 13m longest length and over) that operated on the "cod fishes group[2]" for five different years. In addition to this comes the part of the fishing capacity employed to exploit

[2] The cod fishes group include, the Arcto-Norwegian cod, the Arcto-Norwegian haddock, whiting, Greenland halibut, saith, etc.

other species, say, the "herring fishes group" that are used to land the cod fishes as bycatch.

[Table 2.1 in here!]

Using Norwegian data[3], we calculated the number of coastal fishery vessels and trawlers used by Norwegian fishers in the exploitation of the cod fishes group in 1991 to be about 638 and 58, respectively. These landed about 130 and 270 thousand tonnes of cod, respectively.

To facilitate our analysis three simplifications (about this fishery) are made[4]. First, only Norwegian prices and costs are used in the analysis. Second, the vessel types employed in the exploitation of the resource are grouped into two broad categories, namely, the coastal and the trawl fisheries, and placed under the management of two separate and distinct management authorities, henceforth to be known as Coastal Fisheries Management (C), and Trawl Fisheries Management (T). Third, only the *most cost effective* vessels[5] in each of these categories are assumed to be employed in the exploitation of the resource. The assignment of two separate and distinct fleets to the two management authorities captures, to some extent, the division of the stock between Norway and Russia, but even in Norway a division is usually made between the coastal fleet and the trawlers, and the Norwegian quota is divided between these.

3.0 The model

The model presented here builds on that discussed in Sumaila (1994), to which the reader is referred for details. Here, a two-stage, two-player, dynamic, deterministic, non-cooperative game model is put together, the two players being T and C. By a game we mean a normal (strategic) form, simultaneous-move game in which both players make their investment decisions in ignorance of the decision of the other. At stage one of the game, each player invests in fishing capacity *ex ante*, having in mind that such investment is irreversible. Then in stage two the players employ their chosen capacity investment to exploit the shared resource for the next 15 years, subject to the stock dynamics and nonnegativity constraints.

Both T and C are assumed to be rational and act here to maximize their discounted profit (payoff) function $\Pi_i : K_T \times K_C \rightarrow \Re$, where K_T and K_C are the pure strategy sets of player *i* = T, C, that is, the set of fishing capacity (number of vessels or fleet size) that a player can choose from. Player *i's* payoff at an outcome (k_T, k_C) is then given by $\Pi_i(k_T, k_C)$. A major aim of this modeling exercise is to find the strategy pair (k_T^*, k_C^*) such that no player will find it in his interest to change strategy given that his opponent keeps to his. In other words, we are interested in finding *Nash non-cooperative equilibrium* in a two-player fishery game, where k_T^* is a best reply to k_C^* and vice versa. This is equivalent to stipulating that the inequalities

[3] Data in tables E21 - E51 in Lønnsomhetsundersøkelser (1979 - 1990) were used for the calculations.
[4] See Sumaila (1994) for the justifications for these simplifications.
[5] Cost effectiveness is defined here in terms of least cost per kilogram of fish landed.

$$\Pi_T(k_T^*, k_C^*) \geq \Pi_T(k_T, k_C^*)$$

(1) $$\Pi_C(k_T^*, k_C^*) \geq \Pi_C(k_T^*, k_C)$$

hold for all feasible k_T and k_C.

3.1 On existence of Nash equilibrium

Nash (1950, 1951) proved the existence of equilibrium points under certain assumptions on each player's strategy space and corresponding payoff function. Essentially, he dealt with matrix games. Rosen (1965) went further to show that when every joint strategy lie in a *convex, closed*, and *bounded* region in the product space and each player's payoff function Π_i, i = T, C is *concave* in his own strategy and *continuous* in all variables, then there is at least one Nash equilibrium of the game. This result is stated in theorem 1 below.

THEOREM 1 (Existence of Nash equilibrium, Rosen (1965)): *An equilibrium point exist for every concave n-person game.*

The game we formulate in this paper is a concave 2-person game, and hence satisfies the above theorem. We can therefore expect at least one Nash equilibrium to exist in our game.

3.2 On uniqueness of Nash equilibrium

Two steps are taken here to deal with the vexing problem of equilibrium selection.

Step 1: Only open loop strategies are allowed in the second stage of the game[6]. That is, each player commits, in advance, his fishing capacity to a fixed time function rather than a fixed control law (closed loop strategies). Note that unlike in the case of fixed control laws, where the choice of control depends on the past history of the game, fixed time functions are independent of the actions of the opponent so far in the game. In information theoretic sense open loop corresponds to the receipt of no information during play, while closed loop represents full information.

The main reason we stick to open loop strategies, even though it is not likely to lead us to closed form solutions, is that it would be practically impossible to compute the predictions of our model if closed loop strategies were allowed. This is because closed loop strategies normally entail complex and huge numbers of strategies in repeated games (see Binmore, 1982). Another reason is that the new "Folk Theorem for Dynamic Games" introduced by Gaitsgory and Nitzan (1994), gives us reason to believe that under certain monotonicity assumptions, the set of closed loop solutions that may emerge from our model may coincide with the open loop solutions we compute herein.

---

[6] In a sense one can argue that the game we formulate herein is not a "pure" open loop strategy game. This is because although the fishing capacities are chosen once and for all, the capacity utilization is chosen in each period depending on the stock size.

Step 2: The same shadow prices (that is, Lagrange multipliers) are imposed across both players for resource constraint violation. Flåm (1993) shows that in addition to this, if the marginal profit correspondence is *strictly monotone*, then there exists a unique Nash equilibrium for our game. Incidentally strict monotonicity of marginal profit correspondence is also a sufficient condition for convergence in our model.

In the presentation of the mathematical equations in the rest of the model, two other subscripts (a=0,...,A, and s=1,...,S) are used to denote age groups of fish, and time periods or stages, respectively[7]. Based on the life expectancy of cod, the last age group A, is set equal to 15. The finite time horizon of the game, S, is set equal to 15 due to computational limitations.

Catch

Let catch of age group *a* (in number of fish) by player *i* in fishing period *s*, $h_{i,a,s}$, be given by

$$h_{i,a,s} = q_{i,a} n_{a,s} E_{i,s} \tag{2}$$

where the effort profile, $E_{i,s} = k_i e_{i,s}$, and k_i is the *ex ante* fixed capacity investment of player *i*; $e_{i,s} \in [0,1]$, is the capacity utilisation, that is, the fraction of k_i taken out for fishing in a given year; $n_{a,s}$ is the post catch number of fish of age *a* in fishing period *s* and $q_{i,a}$ is the player and age-dependent catchability coefficient, that is, the share of age group *a* cod being caught by one unit of effort.

Total catch by all players of age group *a* in period *s* can thus be written as

$$h_{a,s} = \sum_i q_{i,a} n_{a,s} k_i e_{i,s} \tag{3}$$

Total catch in weight by all players over all age groups in period *s* is given by

$$h_s = \sum_a h_{a,s} w_a \tag{4}$$

where w_a is the weight of fish of age group *a*.

Costs and Prices

The fishery is assumed to face perfectly elastic demand. Thus, the ex-vessel selling price of fish per kilogram, V, is assumed to be constant. The harvesting costs per vessel employed by player *i* in period *s*, $\Psi_{i,s}$ consist of fixed costs (φ_i) and variable costs (ϑ_i) which are proportionate to $e_{i,s}$,

$$\Psi_{i,s} = \varphi_i + \frac{\vartheta_i}{(1+b)} e_{i,s}^{1+b} \tag{5}$$

[7] Recall that the subscript denoting player is i = T, C.

where $b = 0.01$. This formulation of the cost function ensures strict concavity of individual profit as a function of individual effort. This strictness is important for the sake of convergence.

Revenue

The revenue to player i, in period s, $r_{i,s}$, comes from the sale of his catch over all age groups in that period, that is,

$$r_{i,s} = V \sum_{a} w_a h_{i,a,s} \tag{6}$$

Profits

Player i's profit in a given period s is then given by the equation

$$\pi_{i,s}(n_s, e_{i,s}) = r_{i,s} - k_i \psi_{i,s} \tag{7}$$

where $k = (k_i, k_{-i})$. Note that $\pi_{i,s}$ is a function of the actual fish abundance in a period, n_s, and own effort of a player in that period. We restrict our analysis to the case of perfectly non-malleable capital in which the depreciation rate is equal to zero and capital has a negligible scrap value. Even though this simplification is not quite realistic, the qualitative effect of this is expected to be insignificant. The profit function given by equation (7) is formulated to incorporate this restriction.

Objective

Given k_i, the second stage problem of player i is to find a sequence of capacity utilization, $e_{i,s}$ (s=1,2,...,S), to maximise his present value (PV) of profits (payoff), that is,

$$\Pi_i(k_T, k_C) = \max_{e_i} \sum_{s=1}^{S} \delta_i^s \pi_i(n_s, e_{i,s}) \tag{8}$$

subject to the stock dynamics (see below) and the obvious nonnegativity constraints, expressed mathematically as $e_{i,s} \geq 0$, for all i and s; $n_{a,s} \geq 0$, for all a and s; $n_{a,S+1} \geq 0$, for all a; and $n_{a,0} \geq 0$ given. Here, $\delta_i = (1 + r_i)^{-1}$ is the discount factor; $r_i > 0$ denotes the interest rate of player i; and $n_{a,0}$ is a vector representing the initial number of fish of each age group.

An important but self-evident component of this game is that players are jointly constrained by the population dynamics of the fish stock. Nature is introduced (as a player) in the game with the sole purpose of ensuring that the joint constraints are enforced. The decision variable of nature is thus the stock level - its objective being to ensure the feasibility of the stock dynamics. Formally, nature's objective is expressed as 0 if the stock dynamics is feasible, and $-\infty$ otherwise.

It is worth mentioning here that unless players enjoy bequest, they will typically drive the fishable age groups of the stock to the open access equilibruim level at the end of the game, if the terminal restriction is simply $n_{a,S+1} \geq 0, \ \forall a$. To counteract this tendency, one can exogeneously impose the more restrictive constraint, $n_{a,S+1} \geq \tilde{n}_a$, where $\tilde{n}_a$ is a certain minimum level of the stock of age group *a* that must be in the habitat at the end of period S+1. This is what is done here. Alternatively, this restriction can be imposed endogeneously by obliging the players to enter into a stationary regime maintaining constant catches and keeping escapment fixed from S onwards.

Stock dynamics
Let the stock dynamics of the biomass of fish in numbers $n_{a,s}$, (that is, the joint constraint mentioned above) be described by

$$
\begin{aligned}
& n_{0,s} \leq f(B_{s-1}), \\
& n_{a,s} + h_{a,s} \leq \xi_{a-1} n_{a-1,s-1}, \qquad \text{for} \quad 0 < a < A \\
(9) \qquad & n_{A,s} + h_{A,s} \leq \xi_A n_{A,s-1} + \xi_{A-1} n_{A-1,s-1},
\end{aligned}
$$

where $f\left(B_{s-1} = \frac{\alpha B_{s-1}}{1+\gamma B_{s-1}}\right)$ is the Beverton-Holt recruitment[8] function; $B_{s-1} = \sum_a p_a w_{sa} n_{a,s-1}$ represents the post-catch biomass in numbers; p_a is the proportion of mature fish of age *a*; $w_{s,a}$ is the weight at spawning of fish of age *a*; α[9] and γ are constant parameters chosen to give a maximum stock size of about 6 million tonnes - a number considered to be the approximate carrying capacity of the habitat[10]; ξ_a is the natural survival rate of fish of age *a*; and $h_{a,s}$ defined earlier, denotes the combined harvest of fish of age *a*, in fishing season *s* by all agents.

4.0 Numerical method

An algorithm is developed to resolve the second stage game problem, that is to find the answer to the following question: given the fixed capacity choice of the players in stage one of the game, what level of capacity utilization should they choose in each fishing period so as to maximize their respective economic benefits? For detailed discussions of the theoretical basis for the algorithm see in particular Flåm (1993), and also Sumaila (1993). In what follows, we briefly outline the conceptual algorithm and describe in concrete terms its practical implementation, the detailed problem-specific algorithm is relegated to an appendix.

[8] In this model, recruitment refers to the number of age zero fish that enter the habitat in each fishing period.
[9] $\alpha = f'(0)$, is the number of recruits per unit weight of biomass "at zero" or the polutation level.
[10] Researchers at the Institute of Marine Research, Bergen, estimate the maximum sustainable yield (MSY) stock level to be about 3 million tonnes: With an assumption that the MSY stock level is one half of the pristine stock level we get the figure of 6 million tonnes.

Suppose for illustrative purposes that all constraints (except nonnegativity ones) are incorporated into one concave restriction of the form $\Phi(n, e_i, e_{-i}) \geq 0$, where e_{-i} is the profile of capacity utilisation of *i*'s rival, e_i is the equivalent for player *i*, and *n* is the stock profile (note that $n \in R^{(A+1),S}$, and $e_i \in R^S$ are large vectors, hence, the *a* and *s* subscripts are ignored here). Then we can state the payoff function of player *i* as follows

(10) $$\Pi_i(k) = \Pi_i(k_i, k_{-i}) = \max_{e_i} L_i(n^*, e_i, e^*_{-i}, y^*, k)$$

where

(11) $$L_i(n, e_i, e_{-i}, y, k) = \sum_{s=1}^{S} \delta_i^s (r_{i,s} - k_i \psi_{i,s}) + y\Phi^-(n, e_i, e_{-i}, k)$$

is a modified Lagrangian, y is the Lagrange multiplier, (e_i^*, n^*, y^*) are equilibrium solutions of the variables in question, and Φ^- is given by $\min(0, \Phi)$. The adjustment rules in the algorithm are then given by

(12) $$\dot{e}_i = \frac{\partial L_i(n, e_i, e_{-i}, y, k)}{\partial e_i}, \quad \forall\, i$$

(13) $$\dot{y} = -\frac{\partial L_i(n, e_i, e_{-i}, y, k)}{\partial y} = -\Phi^-$$

(14) $$\dot{n} = y\frac{\partial \Phi^-}{\partial n}$$

where $\frac{\partial L_i(.)}{\partial e_i}$ and $\frac{\partial L_i(.)}{\partial y}$ are the partial derivatives of $L_i(.)$ with respect to e_i and y, and $\frac{\partial \Phi^-}{\partial n}$ is the partial derivative of the constraint function with respect to *n*.

The algorithm then comes in differential form: Starting at arbitrary initial points (e_i, y, n), the dynamics represented by the adjustment rules are pursued all the way to the stationary points (e_i^*, n^*, y^*). Such points satisfy, by definition, the steady-state generalized equation system:

$$0 = \Phi^-(n, e_i, e_{-i}, k),$$

$$0 \in \partial_{e_i}\left[\sum_{1}^{S} \delta_i^s (r_{i,s} - k_i \psi_{i,s}) + y\Phi^-(n, e_i, e_{-i}, k)\right], \quad \forall\, i$$

with $y^* \geq 0$. It is standard from mathematical programming that individual optimality and stock feasibility then obtains. The numerical scheme uses Euler's method to integrate (12), (13), and (14) all the way to the equilibrium solutions $\left(e_i^*, n^*, y^*\right)$.

5.0 Numerical results

A strategic form for our game is given in table 5.1. To obtain these results, the newly developed dynamic simulation software package, POWERSIM[11], is used as computational support. The parameter values listed in table 5.0 are used for the computations. In addition, α and γ are set equal to 1.01 and 1.5, respectively, to give a maximum biomass of 6 million tonnes for a pristine stock. Based on the survival rate of cod, ξ_a is given a value of 0.81. The price parameter $V_,$ is set equal to NOK 6.78. The variable costs (ϑ_T & ϑ_C) and fixed costs (φ_T & φ_C) for engaging a vessel are calculated to be (NOK 12.88 & 0.88) and (NOK 15.12 & 0.65) million for T and C, respectively[12]. The interest rate, r_i, is set equal to 7% as recommended by the Ministry of Finance of Norway. The initial number of cod of each age group is calibrated using the 1992 estimate of the stock size of cod in tonnes[13].

[Table 5.0 in here!]

In table 5.1, rows represent player T's pure strategies k_T and columns represent player C's pure strategies k_C. Player T's payoff is placed in the southwest corner of the cell in a given row and column and the payoff to player C is placed in the northeast corner. The best payoff for player T in each column and the best payoff to player C in each row are bold-faced. As an example notice that 19.4 has been bold-faced in cell $k_T = 65$ and $k_C = 500$, since 19.4 lies in row $k_T = 65$, this tells us that pure strategy $k_T = 65$ is player T's best reply to a choice of pure strategy $k_C = 500$ by player C.

[Table 5.1 in here!]

Notice that the only cell in table 5.1 that has both payoffs bold-faced is that which lies in row $k_T = 57$ and column $k_C = 1050$. Thus the only pure strategy pair that constitutes a Nash equilibrium is (57,1050). Each strategy in this pair is a best reply to the other. This gives total PV of economic benefit equal to $\sum_i \Pi_i(57{,}1050) =$ NOK 25.87 billion, with $\Pi_T(57{,}1050) =$ NOK 11.84 billion and $\Pi_C(57{,}1050) =$ NOK 14.03 billion, respectively.

[Table 5.2 in here!]

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[11] Powersim is a dynamic simulation software package developed by ModellData AS in Bergen, Norway. The model has many powerful features, including the ability to process array variables.

[12] The price per kilogram of NOK 6.78 is taken from table 22 in Central Bureau of Statistics of Norway (1989-1990). The parameters ϑ_i and φ_i are calculated using cost data in Lønnsomhetsundersøkelser (1979 - 1990).

[13] The 1992 stock size is estimated at 1.8 million tonnes (Ressursoversikt, 1993).

The overall PV of economic rents from the fishery as a function of k_T and k_C are given in table 5.2. The entries in each cell of this table are simply the sum of the entries in corresponding cells in table 5.1. These results indicate an optimal fleet consisting of 1100 coastal vessels and no trawlers with PV of economic rent equal to NOK 36.11 billion, or NOK 32.83 million per vessel. In contrast, a discounted profit maximising fleet consisting solely of trawlers would be made up of 70 vessels and earn a PV of economic rent of NOK 32.42 billion, or NOK 463.14 million per vessel.

The model thus appears to support the general theoretical assertion that non-cooperation generally results in rent dissipation through the use of excess fishing capacity. The economic theory of fisheries predicts that in an open access fishery, economic rent would normally be dissipated completely (Gordon, 1954; Hannesson, 1993). Table 5.2 indicates that a trawl-coastal fishery vessel combination of a little over 120-2500 vessels would dissipate discounted economic rent from the fishery to nil. Thus, this vessel combination or its equivalent, is our models prediction of the open access fishing capacity. If the agents in the fishery were to receive subsidies totally NOK 9.48 billion (in present value) in addition to having open access to the resource, then the model's prediction of fishing capacity is 140 trawlers and 3000 coastal vessels or their equivalent. It should be interesting to compare the equilibrium stock and harvest profiles that would result under "open access plus subsidy", open access, Nash non-cooperative, and the sole ownership equilibria. This is done graphically in figures 5.1 and 5.2 below. The figures illustrate clearly the adverse effects of "open access plus subsidy", open access, Nash non-cooperative equilibria as compared to the optimal solution.

[Figures 5.1 and 5.2 in here!]

Notice that contrary to expectations, the biomass is not completely depleted at the end of the game. There are two possible reasons for this. In the first place players are not allowed to exploit all age groups of fish. Secondly, even if this was allowed, it would not be economically profitable to catch every single fish available.

5.1 Perfect malleability versus perfect non-malleability

We label the model in Sumaila (1994) the perfect malleable capital model and that in this paper the perfect non-malleable capital model. The purpose here is to compare the capacity investments and the PV of economic rent accruable to the players both individually and to society at large, in the two models. To do this, the perfect malleable model is run using comparable prices and costs. Table 5.3 below gives the capacity predictions of the two models, and the PV of economic rents to the players when both agents are active; when only T is active; and when only C is active. It is seen from this table that vessel capacity investment varies from year to year in the case of the malleable capital model.

[Table 5.3 in here!]

A possible interpretation here is that each player evaluates his optimal capacity requirement in a given year and then rents precisely this quantity from a rental firm for fishing vessels[14]. For instance, when both are active, T's optimal vessel size varies from a high of 65 in the third year to a low of 41 trawlers in year 15, while C's varies from a high of 939 in year 3 to a low of 564 in the last year. In the non-malleable capital model, however, T's *ex ante* fixed capacity investment is 57 trawlers, and the corresponding capacity investment for C is 1050 coastal vessels.

The economic results given by the two models are given in the third column of table 5.3. Two important observations can be made from this table. First, the maximum economic rent from the resource are different in the two models: NOK 44.53 billion is achieved in the malleable capital model and NOK 36.11 in the non-malleable one. The higher PV of economic rent achievable in the malleable capital model can be attributed to the removal of the restriction that non-malleability of capital imposes on the agents. The negative impact of this restriction is quite substantial, reaching upto about NOK 12 billion (or 47% of what is achievable under the restriction) in the case of the Nash equilibrium solution. This clearly demonstrates that there is much to be gained from establishing rental firms for fishing vessels. Or rather, by allowing mobility of vessels between different stocks[15].

Second, in both models the best economic results are achieved when C operates the fishery alone. However, the superiority of C becomes sharper in the non-malleable capital model: There is a difference of well over 10% (in favour of C) in the PV of economic rent accruable in the non-malleable capital model. In the malleable capital model, however, a difference of only about 5% is noted. This finding may have to do with the relatively high fixed cost of trawlers and the fact that fixed costs must be taken as given by the players in the non-malleable capital model.

5.2 Comparison with available data on the Arcto-Norwegian cod fishery

It is stated in section 2 that in 1991 the equivalent of fishing capacity of about 638 coastal vessels and 58 trawlers were operated by Norwegian fishers to land 130 and 270 thousand tonnes of cod, respectively. This implies that catch per trawler is about 4655 tonnes and catch per coastal fishery vessel is 203 tonnes. Now, the Nash equilibrium strategies stipulate vessel sizes of 1050 for C and 57 for T. Together, these capacities are used to land an average of about 842 thousand tonnes a year[16]. Of the total, trawlers land 412 thousand tonnes and the coastal fishery vessels 430 thousand tonnes. Thus catch per vessel are 7228 and 410 tonnes for a trawler and coastal fishery vessel, respectively. These numbers signify the incidence of overcapacity in the Arcto-Norwegian cod fishery even in comparison to the results from a non-cooperative solution. Comparison with the sole owner's optimal solution reveals an even greater degree of overcapacity in the fishery: In this case 1100 vessels are used to land on average about 770 thousand tonnes of fish per year by C (that is, 700 tonnes per vessel) and 70 trawlers to land an average of about 780 thousand

[14] A practical way to view these variations is that the agents in this model have alternative uses for their vessel capacities, thereby making it possible for them to divert excess capacity in any given year to such uses.

[15] Hannesson (1993) looks more closely at the possible gains from allowing mobility of vessels between different stocks.

[16] Note that these harvests comprise both Norwegian and Russian landings, since we do not differentiate between the two in our model.

tonnes per year by T (well over 10000 tonnes per vessel). A catch per trawl vessel of 10000 tonnes per year appears to be high, probably the proportionality assumption (underlying the harvest function) between the stock size and the catch per vessel is appropriate only when variation in the stock size is not too large.

5.3 Effect of fixed costs, interest rates, initial stock size, and terminal constraint

Fixed costs Sensitivity analysis shows that (for the game solution), the elasticity of the PV of economic benefits accruable to the players T and C with respect to fixed costs are about -0.7 and -0.4, respectively[17]. Hence, to achieve the higher discounted economic rent, T needs a drop in its fixed costs relative to those of C of about 28%. The equivalent elasticities in the sole owner solutions are -0.3 for T and -0.2 for C, which implies that T needs a drop in its fixed costs relative to those of C of about 39% to take over from C as the producer of the optimal solution. We also investigated the effects of zero fixed costs. This is the same as assuming that fixed costs are considered to be "sunk" by the agents. Under such an assumption, C and T achieve discounted economic rents of NOK 20.27 and 19.7 billion, respectively, in the game situation, and NOK 42.06 (C) and 42.64 (T) in the sole ownership solutions. We see that the previously clear superiority of C is now neutralised to a great extent.

Interest rates For the game situation, we found that a 1% drop in the interest rate faced by both players results in a 1% increase in the relative profitability[18] of T. On the other hand, an equivalent drop in the interest rate in the sole ownership scenario leads to an increase of 0.5% in the relative profitability of C. Intuitively, it is not difficult to understand why T does relatively better in the game situation while C does relatively better in the sole ownership case: Since T harvests everything from age group 4 and above, while C harvests only age groups 7 and above, it is no wonder that T is the one best positioned to capitalize on the increase in patience that a decrease in interest rate entails. C does better in the sole ownership scenario because an increase in patience plus the fact that C harvests fish from age group 7 and above means that a larger proportion of the stock will reach maximum weight before they are harvested, thereby resulting in better relative profitability for C.

Initial stock size To investigate the effect of the initial stock size on the relative profitability of the agents, the model is re-run with 50% and 150% of the base stock size of 1.8 million tonnes. The results we obtained indicate that in the sole ownership solutions, T improves it's relative profitability as the stock size increases; from 86.85% when the stock size is only 50% of the base case to 92.1% when the stock size is 150% of the base case. The effect of stock size on the relative profitability of the agents in the game solution is however not that clear: T increases it's relative profitability both when the stock size is only 50% of the original (90.6% as against 84.4% in the base case) and when the stock size is 150% of the base stock size. In this case T's relative profitability is 92.3%. As these numbers show T's relative

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[17] Recall that the elasticity of a function, f(x,y), with respect to x is defined as the percentage increase in f(x,y) resulting from a 1% increase in x.

[18] Relative profitability is defined as discounted economic rent to T divided by discounted economic rent to C multiplied by 100.

profitability increases by a larger margin when the stock size increases than when it decreases.

Terminal constraint on the stock size to be left behind at the end of the game A requirement that not less that 50% of the initial stock size should be left in the sea at the end of the game changes the outcome of the game significantly in the game situation. In the sole ownerhip situation, however, the same solutions as in the base case are obtained, mainly because this constraint is not binding in these cases. Under such a requirement it turns out that T comes out better (in contrast to the base case where it is C that does better) , earning NOK 15.97 billion as against C's NOK 12.98 billion. An important point to note here is that the introduction of the terminal constraint, in the game environment, leads to an increase in the overall benefit from the fishery from NOK 25.87 to NOK 28.95 billion. This explains why economists advocate regulation when common property resources are exploited in non-cooperative environments.

6.0 Concluding remarks

The main findings of this study can be stated as follows: The optimal capacity investments in terms of number of vessels for T and C in a competitive, non-cooperative environment are 57 trawlers and 1050 coastal vessels, respectively. The use of these capacities results in discounted benefits of NOK 11.84 and 14.03 billion, respectively, to T and C, and an overall discounted economic benefit of NOK 25.85 billion to society at large. Using only T and C vessels in the exploitation of the resource, the optimal fleet sizes are 70 and 1100, respectively. In these cases the PV of economic rents are NOK 32.42 and 36.11 for T and C. We also found out that, as expected, the results obtained are rather sensitive to perturbations in fixed costs, interest rates, initial stock size and the terminal constraint.

It is in order to state here that given that modeling and computation are always exercises in successive approximation (Clark & Kirkwood, 1979), our estimates should not be taken too literally. Having said this, the results of the analysis indicate that in it's current state the Arcto-Norwegian cod fishery appears to suffer from over capacity. Hence, the practical implication of this study with respect to efficient management of the resource is that the excess capacity should be run down as rapidly as possible to a certain level[19] somewhere above the Nash equilibrium capacity level. Thereafter the remaining excess capacity is allowed to depreciate to the "desired" level by itself. From then on new capacity investment is undertaken only to make up for depreciation. This would ensure each player his best possible outcome and the society the second best solution.

The practical implication of the results obtained would have been somewhat different if the fishery were under-exploited. In this case, we distinguish between starting a completely new fishery and the case where fishing is currently in progress. In the case of a new fishery, there is a real possibility for realising the first best solution by allowing only C to exploit the resource with a capacity size of about 1100 coastal

[19] The choice of this level will depend on both the depreciation rate and the difference between the cost of acquiring a new vessel and the disinvestment resale price of the vessels relative to the price of new vessels.

fishery vessels. However, if political, social and cultural realities dictate the participation of both players and this were to be in a non-cooperative environment, then each player should aspire to start off with its Nash equilibrium capacity size as computed herein.

Appendix

The algorithm. The following non-standard Lagrangian function for player *i* follows from our model:

$$L_i(n,e,y,k) = \sum_{s=1}^{S} \left[\begin{array}{l} \delta_i^s\left(r_{i,s} - k_i\psi_{i,s}\right) + y_{0,s}\left(f(B_{s-1}) - n_{0,s}\right)^- \\ + y_{A,s}(\xi_A n_{A,s-1} + \xi_{A-1} n_{A-1,s-1} - n_{A,s} - h_{A,s})^- \\ + \sum_{a=1}^{A-1} y_{a,t}(\xi_{a-1} n_{a-1,s-1} - n_{a,s} - h_{a,s})^- \end{array} \right]$$

where $y := y_{a,s}$ is the player-invariant, but age and season-variant multipliers, and all other variables are as defined earlier.

The negative superscript on some components of the equation above is a device introduced to ensure monotone convergence of multipliers by focusing attention on the situations where there are constraint violations. Such a device results in multipliers that are different from those associated with the classical Lagrangians. There is a relationship between the two kinds of multipliers, however, the exact relationships are not so easy to retrieve. It is therefore necessary, at this juncture, to call for caution when interpreting the computed equilibrium multiplier levels.

The gradient information obtainable from $L_i(n,e,y)$, gives the adjustment equations for the effort levels and multipliers. We first introduce a special (switch) function related to the derivative of Φ^-, before we state the adjustment equations. Let the function $H(r) = 1$ if $r<0$, and $H(r) = 0$ otherwise. If $r \geq 0$ were a constraint inequality, then H(r) will attain a value of 1 if the constraint is violated, otherwise it attains a value of 0. In writing the adjustment equations below, this switch function is used.

Starting at arbitrary initial guesses of $y_{a,t}$, $n_{a,t}$ and $e_{i,t}$, we pursue the dynamics given by the adjustment equations below, all the way to the equilibrium solutions.

Effort adjustment. The adjustment equation for effort given by $\dfrac{\partial L_i(.)}{\partial e_{i,s}}$ is

$$\begin{aligned} e_{i,s} = {} & \delta_i^s\Big(\sum_a V_i w_a q_{i,a} k_i n_{a,s} - k_i \vartheta_i e_{i,s}^b\Big) \\ & + \sum_{a=1}^{A-1} y_{a,s} H(\xi_{a-1} n_{a-1,s-1} - n_{a,s} - h_{a,s})(-q_{i,a} k_i n_{a,s}) \\ & + y_{A,s} H(\xi_A n_{A,s-1} + \xi_{A-1} n_{A-1,s-1} - n_{A,s} - h_{A,s})(-q_{i,A} k_i n_{A,s}) \end{aligned}$$

Multiplier adjustment. The equations for the sequential adjustment of the multipliers are obtained by taking the negative of the partial differential of L_i with respect to the appropriate multiplier, that is, they come from $-\dfrac{\partial L_i(.)}{\partial y_{a,s}}$.

For age group zero fish, the multiplier is adjusted according to the equation

$$\dot{y}_{0,s} = -H(f(B_{s-1}) - n_{0,s})(f(B_{s-1}) - n_{0,s})$$

Multipliers for fish of age groups between 1 and A-1 are adjusted as follows

$$\dot{y}_{a,s} = -H(\xi_{a-1}n_{a-1,s-1} - n_{a,s} - h_{a,s})(\xi_{a-1}n_{a-1,s-1} - n_{a,s} - h_{a,s})$$

For the last age group, multiplier adjustment is according to

$$\dot{y}_{A,s} = -H(\xi_A n_{A,s-1} + \xi_{A-1}n_{A-1,s-1} - n_{A,s} - h_{A,s})(\xi_A n_{A,s-1} + \xi_{A-1}n_{A-1,s-1} - n_{A,s} - h_{A,s})$$

Here, the RHS of the equations are calculated and then the corresponding multipliers adjusted according to the magnitude and direction of the calculated result.

Nature's adjustment of the stock level. Natures objective can be expressed as

$$\begin{aligned} L_N = & \; y_{0,s}\left(f(B_{s-1}) - n_{0,s}\right)^- \\ & + \sum_{a=1}^{A-1} y_{a,s}(\xi_{a-1}n_{a-1,s-1} - n_{a,s} - h_{a,s})^- \\ & + y_{A,s}(\xi_A n_{A,s-1} + \xi_{A-1}n_{A-1,s-1} - n_{A,s} - h_{A,s})^- \end{aligned}$$

This equation is derived from the fact that once the stock dynamics is obeyed, nature's net benefit is 0, hence, L_N consist of only the constraint equations.

The updating rules for age groups, a=0, a=1,...,A-2, a=A-1, and a=A are different and are given below separately. These are obtained by partially differentiating $L_N(.)$ with respect to the corresponding stock level. That is, they come from $\frac{\partial L_N(.)}{\partial n_{ai,s}}$.

(1) The stock level of age zero fish is adjusted sequentially in accordance with the equation,

$$\begin{aligned} \dot{n}_{0,s} = & \; y_{0,s+1}H(f(B_s) - n_{0,s+1})f'(B_s)\frac{\partial B_s}{\partial n_{0,s}} \\ & - y_{0,s}H(f(B_{s-1}) - n_{0,s}) \\ & + y_{1,s+1}H(\xi_0 n_{0,t} - n_{1,s+1} - h_{1,s+1})\xi_0 \end{aligned}$$

(2) Fish of age groups between 1 and A-2 are updated as follows,

$$\dot{n}_{a,s} = y_{0,s+1} H(f(B_s) - n_{0,s+1}) f'(B_s) \frac{\partial B_s}{\partial n_{a,s}}$$
$$+ y_{a,s} H(\xi_{a-1} n_{a-1,s-1} - n_{a,s} - h_{a,s})(-1 - \sum_i q_{i,a} k_i e_{i,s})$$
$$+ y_{a+1,s+1} H(\xi_a n_{a,s} - n_{a+1,s+1} - h_{a+1,s+1}) \xi_a$$

(3) The last but one age group of fish (i.e., the A-1 age group) is adjusted in accordance to the equation,

$$\dot{n}_{A-1,s} = y_{0,s+1} H(f(B_s) - n_{0,s+1}) f'(B_s) \frac{\partial B_s}{\partial n_{A-1,s}}$$
$$+ y_{A-1,s} H(\xi_{A-2} n_{A-2,s-1} - n_{A-1,s} - h_{A-1,s})(-1 - \sum_p q_{i,A-1} k_i e_{i,s})$$
$$+ y_{A,s+1} H(\xi_A n_{A,s} + \xi_{A-1} n_{A-1,s} - n_{A,s+1} - h_{A,s+1}) \xi_{A-1}$$

(4) Finally, the last age group, is updated using the following equation,

$$\dot{n}_{A,s} = y_{0,s+1} H(f(B_s) - n_{0,s+1}) f'(B_s) \frac{\partial B_s}{\partial n_{A,s}}$$
$$+ y_{A,s+1} H(\xi_A n_{A,s} + \xi_{A-1} n_{A-1,s} - n_{A,s+1} - h_{A,s+1}) \xi_A$$
$$+ y_{A,s} H(\xi_A n_{A,s-1} + \xi_{A-1} n_{A-1,s-1} - n_{A,s} - h_{A,s})(-1 - \sum_i q_{i,A} k_i e_{i,s})$$

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Table 2.1: No. of Norwegian vessels operating on the "cod fishes group" for five different years.

Year	Trawlers	Coastal vessels
1991	57	562
1990	51	572
1988	84	661
1986	118	628
1984	128	718

Table 5.0: Values of parameters used in the model

Age a (years)	Selectivity q(p,a) p=1	p=2	Weight at spawning w(s,a) (kg)	Weight in catch w(a) (kg)	Initial numbers (millions)
0	0	0	0.090	0.10	167.0
1	0	0	0.270	0.30	135.0
2	0	0	0.540	0,6	108.0
3	0	0	0.900	1.00	88.3
4	0.0074	0	1.260	1.40	71.7
5	0.0074	0	1.647	1.83	58.3
6	0.0074	0	2.034	2.26	46.7
7	0.0074	0.00593	2.943	3.27	38.3
8	0.0074	0.00593	3.843	4.27	30.8
9	0.0074	0.00593	5.202	5.78	0,25
10	0.0074	0.00593	7.164	7.96	20.3
11	0.0074	0.00593	8.811	9.79	16.7
12	0.0074	0.00593	1.0377	11.53	13.3
13	0.0074	0.00593	12.456	13.84	10.8
14	0.0074	0.00593	13.716	15.24	8.67
15	0.0074	0.00593	14.706	16.34	7.0

Note: (1) The values for q(p,a) are calculated using the procedure outlined in Sumaila (1994). (2) Player T exploits fish of age 4 and above and player C fish of age group 7 and above (Hannesson, 1993).

Table 5.1: The bimarix game. Gives the payoff to each player as a function of k1 (no. of T vessels) and k2 (no. of C vessels) in billions of NOK. Player T's payoff is placed in the southeast corner of the cell in a given row and column, and the payoff to player C is placed in the northeast corner.

k1	k2 (No. of C vessels)											
(No. of T vessels)	0	500	700	900	1000	1050	1100	1200	1500	2000	2500	3000
		28,22	31,76	35,34	35,99	35,79	**36,11**	35,3	33,89	28,63	26,67	20,87
0		0	0	0	0	0	0	0	0	0	0	0
	0	21,28	24,58	**25,53**	25,28	25,49	25,44	25,04	23,99	24,21	18,5	14,07
20	17,52	13,3	11,32	9,51	8,67	8,38	8,09	7,46	6,08	4,31	3,54	2,66
	0	15,54	17,1	17,7	17,67	**18,13**	17,99	17,89	17,12	14,47	13,3	9,85
40	28,72	18,8	15,29	12,75	11,65	11,39	10,92	10,07	8,14	5,64	**4,48**	**3,19**
	0	14,18	15,66	16,3	16,28	16,54	**16,58**	**16,58**	15,86	13,94	12,08	8,6
45	29,64	19,1	15,57	13,02	11,9	11,57	11,11	10,35	8,22	**5,82**	4,36	2,97
	0	12,92	14,48	15,02	15,09	**15,42**	15,28	15,4	14,66	12,99	11,4	7,54
50	30,57	19,25	15,89	13,23	12,04	11,78	11,19	**10,47**	8,23	5,74	4,27	2,66
	0	11,84	13,35	14,01	14,1	14,34	**14,36**	14,22	13,64	11,8	10,95	6,81
55	31	19,22	15,89	13,25	12,12	11,8	11,29	10,4	8,24	5,6	4,22	2,45
	0	11,48	12,98	13,66	13,66	$\boxed{\textbf{14,03}}$	13,99	13,76	13,54	11,73	10,46	6,45
57	31,45	19,2	15,97	12,1	12,1	$\boxed{\textbf{11,84}}$	**11,37**	10,36	**8,32**	5,59	4,34	2,26
	0	10,87	12,43	13,18	13,24	13,26	**13,42**	13,33	12,89	11,25	9,7	5,81
60	31,57	19,08	15,96	**13,42**	12,2	11,67	11,31	10,36	8,19	5,46	3,81	2,15
	0	10,29	11,74	12,3	12,7	12,67	12,66	**12,75**	12,11	10,81	8,81	6,01
65	32,17	**19,4**	**16**	13,3	**12,31**	11,74	11,29	10,42	8,04	5,35	3,6	1,95
	0	9,41	10,82	11,52	11,58	11,92	**11,95**	**11,95**	11,72	9,83	8,3	4,77
70	**32,42**	18,9	15,65	13,13	11,86	11,63	11,2	10,24	8,04	4,92	3,38	1,41
	0	8,13	9,46	9,88	10,26	**10,67**	10,56	**10,67**	10,39	8,24	7,47	3,58
80	31,7	18,2	15,06	12,24	11,39	11,34	10,67	9,83	7,54	4,17	2,71	0,54
	0	7,33	8,28	9,11	9,4	9,33	**9,63**	9,12	9,35	7,16	6,31	2,03
90	30,05	17,7	14,17	12,02	11,12	10,45	10,32	8,82	6,97	3,41	1,94	-0,63
	0	6,48	7,63	8,2	8,64	**8,88**	8,53	8,66	8,37	6,43	4,93	0,97
100	28,23	16,69	13,84	11,41	10,57	10,28	9,33	8,61	6,08	2,51	0,69	-1,65
	0	5,02	5,91	6,48	6,36	**6,56**	6,51	6,35	6,11	4,09	2,57	-0,69
120	23,5	14,03	11,24	9,1	7,73	7,38	6,84	5,79	3,62	0,15	-1,84	-3,89
	0	4,42	5,35	5,66	5,41	**5,98**	5,52	5,59	5,16	2,59	0,17	-2,74
140	21,63	12,89	10,42	7,81	5,99	5,66	5,08	4,42	1,99	-2,18	-4,58	-6,73

Table 5.2: Overall PV of economic rent from the fishery as a function of k1 (no. of T vessels) and k2 (no. of C vessels), in billions of NOK.

k1	k2 (No. of CFM vessels)										
(No. of TF vessels)	0	500	700	900	1000	1050	1100	1200	1500	2000	2500	3000
0		28,2	31,8	35,3	36	35,8	36,1	35,3	33,9	28,6	26,7	20,87
20	17,5	34,6	35,9	35	34	33,9	33,5	32,5	30,1	28,5	22,1	16,73
40	28,7	34,3	35,4	30,5	29,3	29,5	28,9	27,9	25,3	20,1	17,8	13,04
45	29,6	33,3	31,2	29,3	28,2	28,1	27,7	26,9	24,1	19,8	16,4	11,57
50	30,6	32,2	30,4	28,2	27,1	27,2	26,5	25,9	22,9	18,7	15,6	10,2
55	31	31,1	29,2	27,3	26,2	26,1	25,7	24,6	21,9	17,4	15,2	9,26
57	31,4	30,7	29	26,8	25,8	25,9	25,4	24,1	21,8	17,3	14,8	8,71
60	31,6	30	28,4	26,6	25,4	24,9	24,7	23,7	21,1	16,7	13,5	7,95
65	32,2	29,7	27,8	25,6	25	24,4	24	23,2	20,2	16,2	12,4	7,96
70	32,4	28,3	26,5	24,6	23,4	23,6	23,2	22,2	19,8	14,7	11,7	6,18
80	31,7	26,3	24,5	22,1	21,7	22	21,2	20,5	17,9	12,4	10,2	4,13
90	30	25	22,5	21,1	20,5	19,8	20	17,9	16,3	10,6	8,3	1,4
100	28,2	23	21,5	19,6	19,2	19,2	17,9	17,3	14,5	8,9	5,6	-0,68
120	23,5	19,1	17,2	15,6	14,1	13,9	13,4	12,1	9,7	4,2	0,73	-4,59
140	21,6	17,3	15,8	13,5	11,4	11,6	10,5	10	7,2	0,4	-4,4	-9,48

Table 5.3: Malleable vs. Non-malleable capital. Gives the equilibrium vessel sizes and the overall discounted economic rent that accrues to society from the resource.

	Vessel size (in numbers)		PV of economic benefit (in billion NOK)	
	Malleable	Non-malleable	Malleable	Non-malleable
Both active	(65 -41; 939 - 564)	(57;1050)	38.26	25.87
T active	(80 -49)	(70)	42.35	32.42
C active	(1153 - 761)	(1100)	44.53	36.11

Fig 5.1: Stock profiles (in million tonnes): Illustrates the post-catch stock size in each period for open access plus subsidy (OAS), open access (OA), Nash equilibrium (NE), T only and C only (the optimal solution).

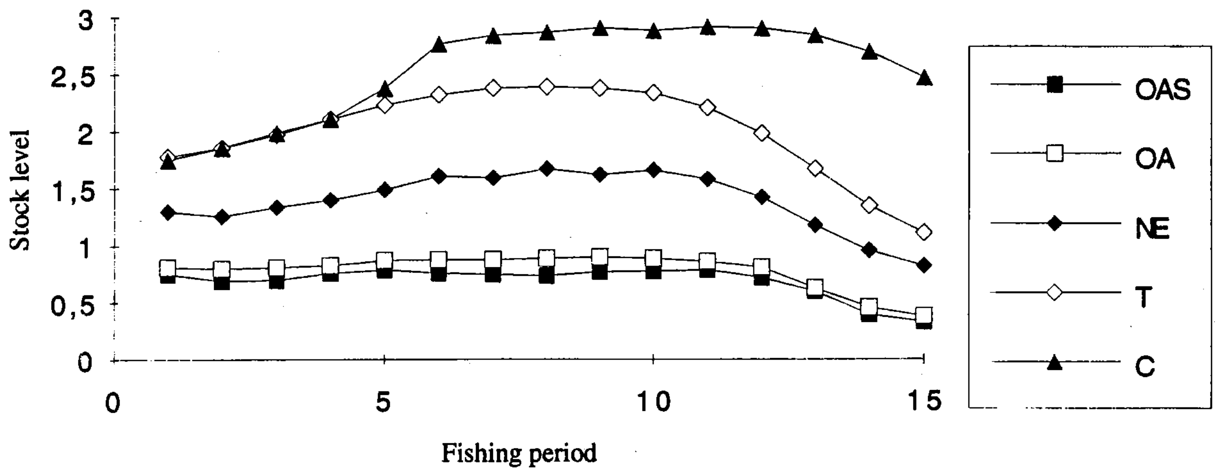
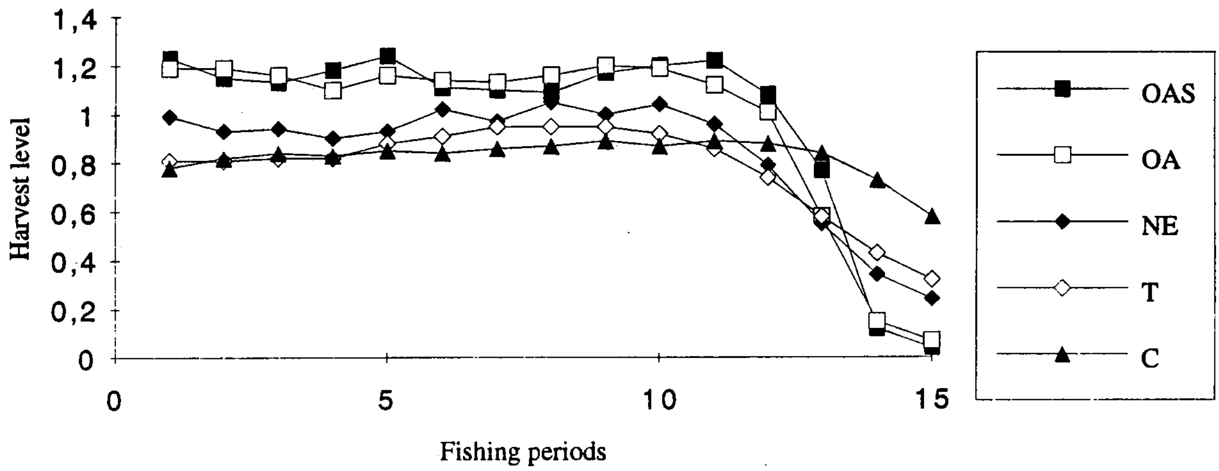


Fig. 5.2: Harvest profiles (in million tonnes): Illustrates total harvest in each period for open access plus subsidy (OAS), open access (OA), Nash equilibrium (NE), T only and C only (the optimal solution).



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